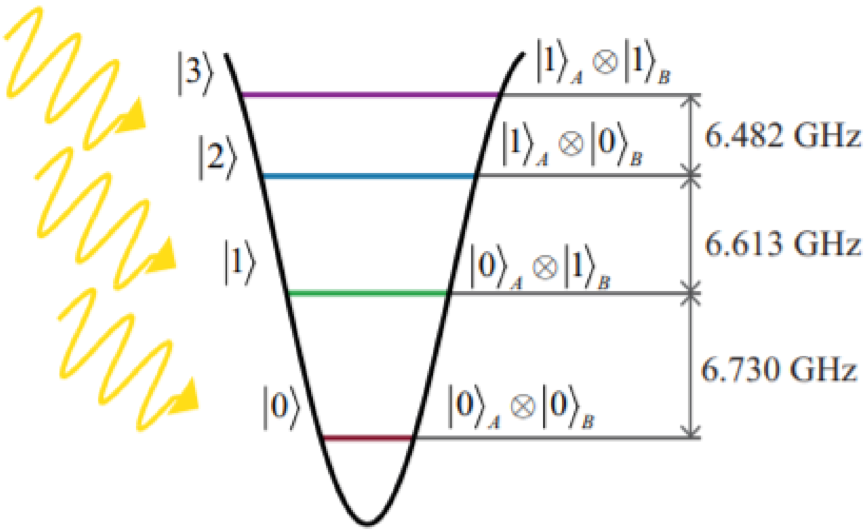
Квантовые вычисления на кудитах

Qudit

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \cdots + \alpha_{d-1}|d-1\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{d-1} \end{pmatrix}$$

$$|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 + \cdots + |\alpha_{d-1}|^2 = 1$$

$d > 2$ – размерность гильбертова пространства $\mathscr{H}_d$



E.O. Kiktenko, A.K. Fedorov, O.V. Man'ko, and V.I. Man'ko. Physical Review A 91, 042312 (2015).

$d = 8 \Leftrightarrow 3$ кубита

1 кубит $d = 5 \Leftrightarrow$ квантовые вычисления

E.O. Kiktenko, A.K. Fedorov, A.A. Strakhov, and V.I. Man'ko. Single qudit realization of the Deutsch algorithm using superconducting many-level quantum circuits // Physics Letters A 379, 1409–1413 (2015).

2 кубита $d = 32 \Leftrightarrow GHZ$ состояния

Imany P. et al. High-dimensional optical quantum logic in large operational spaces //npj Quantum Information. – 2019. – Т. 5. – №. 1. – С. 1-10.

Физические реализации кудитов

- 2 перепутанных кубита
- N-уровневые атомы
- Поляризационные кудиты
- Частотное кодирование
- Временнное кодирование

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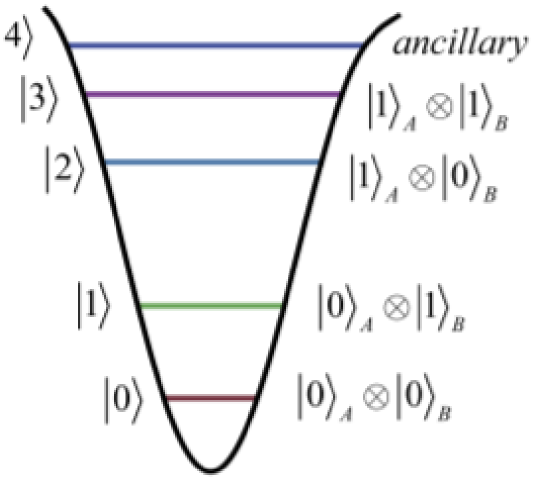

Физические реализации кудитов

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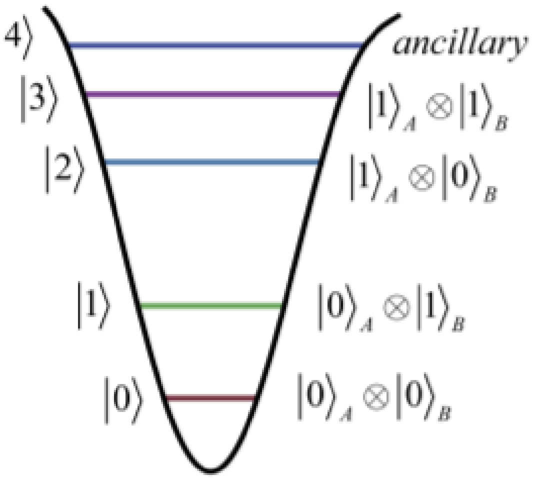

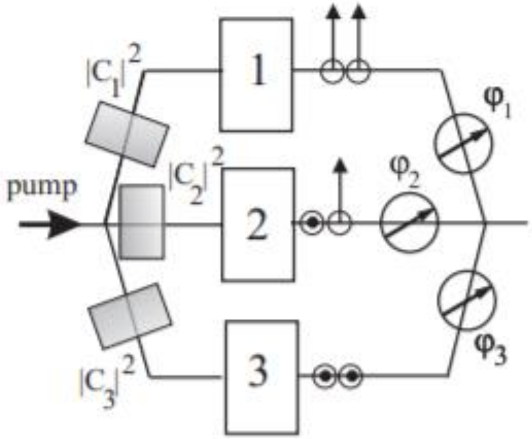

Bogdanov Y. I. et al.
Physical review letters. 2004. V. 93. №. 23. P. 230503.

$$|c\rangle = c_1|2_H, 0_V\rangle + c_2|1_H, 1_V\rangle + c_3|0_H, 2_V\rangle$$

Физические реализации кудитов

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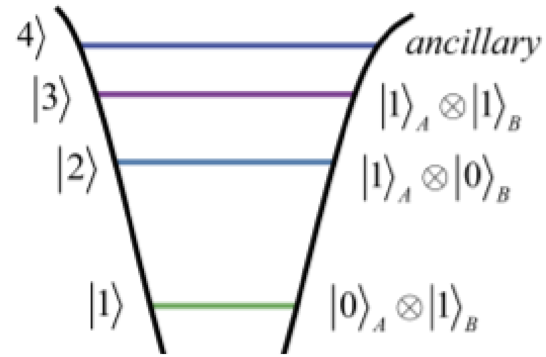

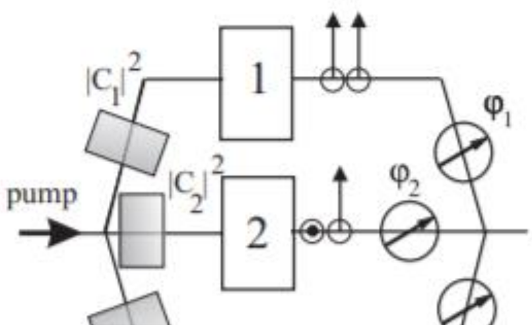

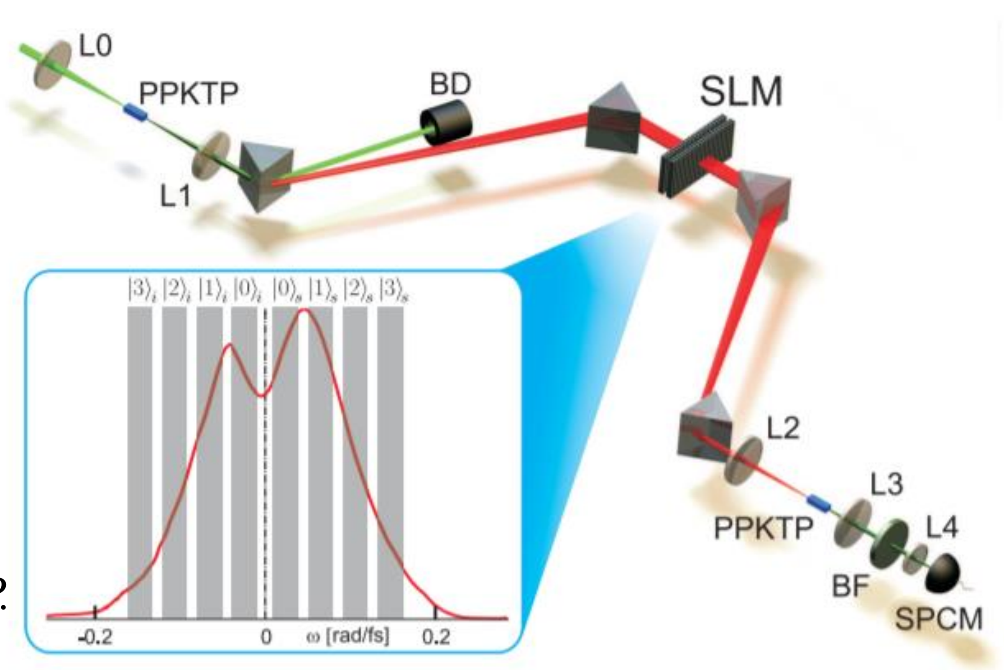

Bernhard C. et al.
Physical Review A. 2013. V. 88. №. 3. P. 032322.

Физические реализации кудитов

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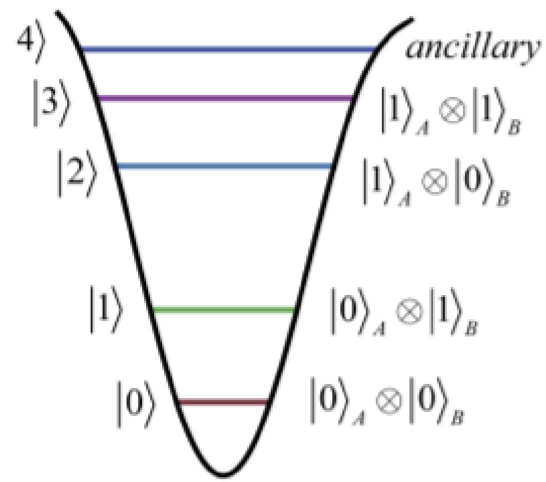

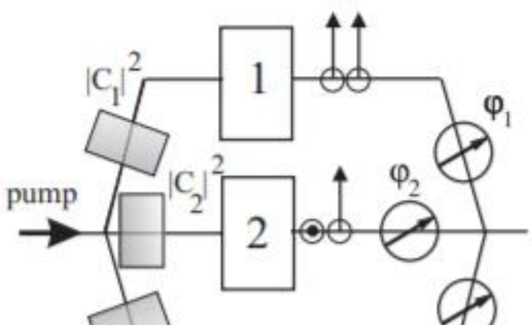

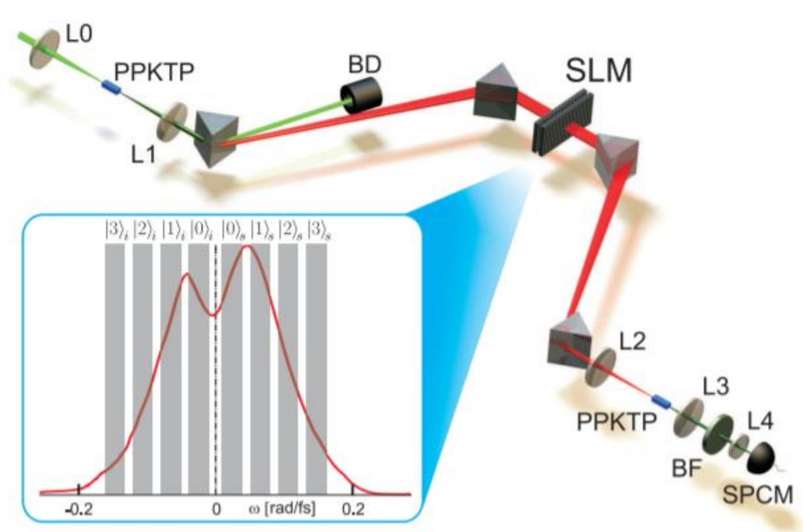

Bessire B. et al. // New journal of physics. 2014. V. 16. №. 3. P. 033017.

Критерий универсальности

Универсальный набор квантовых вентилей $U_k \in U(d^n)$

$$\forall\, U \in H_d^{\otimes n}$$

PHYSICAL REVIEW A, VOLUME 62, 052309

Multivalued logic gates for quantum computation

Ashok Muthukrishnan* and C. R. Stroud, Jr.
The Institute of Optics, University of Rochester, Rochester, New York 14627
(Received 11 February 2000; published 16 October 2000)

We develop a multivalued logic for quantum computing for use in multi-level quantum systems, and discuss the practical advantages of this approach for scaling up a quantum computer. Generalizing the methods of binary quantum logic, we establish that arbitrary unitary operations on any number of d-level systems ($d > 2$) can be decomposed into logic gates that operate on only two systems at a time. We show that such multivalued logic gates are experimentally feasible in the context of the linear ion trap scheme for quantum computing. By using d levels in each ion in this scheme, we reduce the number of ions needed for a computation by a factor of $\log_2 d$.

Критерий универсальности

Универсальный набор квантовых вентилей $U_k \in U(d^n)$

$\forall\, U \in H_d^{\otimes n}$

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tation by a factor of $\log_2 d$.

PHYSICAL REVIEW A **71**, 052318 (2005)

Criteria for exact qudit universality

Gavin K. Brennen,[1,*] Dianne P. O'Leary,[2,3,†] and Stephen S. Bullock[3,‡]

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[2]*University of Maryland, Department of Computer Science, Collge Park, Maryland 20742, USA*
[3]*National Institute of Standards and Technology, Mathematical and Computational Sciences Division, Gaithersburg, Maryland 20899-8910, USA*

(Received 1 September 2004; revised manuscript received 21 December 2004; published 16 May 2005)

We describe criteria for implementation of quantum computation in qudits. A qudit is a d-dimensional system whose Hilbert space is spanned by states $|0\rangle, |1\rangle, \ldots, |d-1\rangle$. An important earlier work [A. Muthukrishnan and C.R. Stroud, Jr., Phys. Rev. A **62**, 052309 (2000)] describes how to exactly simulate an arbitrary unitary on multiple qudits using a $2d-1$ parameter family of single qudit and two qudit gates. That technique is based on the spectral decomposition of unitaries. Here we generalize this argument to show that exact universality follows given a discrete set of single qudit Hamiltonians and one two-qudit Hamiltonian. The technique is related to the QR-matrix decomposition of numerical linear algebra. We consider a generic physical system in which the single qudit Hamiltonians are a small collection of $H_{jk}^{x}=\hbar\Omega(|k\rangle\langle j|+|j\rangle\langle k|)$ and $H_{jk}^{y}=\hbar\Omega(i|k\rangle\langle j|-i|j\rangle\langle k|)$. A coupling graph results taking nodes $0, \ldots, d-1$ and edges $j \leftrightarrow k$ iff $H_{jk}^{x,y}$ are allowed Hamiltonians. One qudit exact universality follows iff this graph is connected, and complete universality results if the two-qudit Hamiltonian $H=\hbar\Omega|d-1,d-1\rangle\langle d-1,d-1|$ is also allowed. We discuss implementation in the eight dimensional ground electronic states of ^{87}Rb and construct an optimal gate sequence using Raman laser pulses.

Пример набора универсальных квантовых вентилей

$$\Gamma_d := \left\{ X_d^{(l)}, Z_d, C_2\left[R_d\right] \right\}$$

Luo, M, and Wang, X. Universal quantum computation with qudits. Sci China Phys Mech Astron (2014). 57:1712–7.

$$X_j(x,y) = \begin{pmatrix} I_{j-1} & & & \\ & \frac{x}{\sqrt{|x|^2+|y|^2}} & \frac{-y}{\sqrt{|x|^2+|y|^2}} & \\ & \frac{y^*}{\sqrt{|x|^2+|y|^2}} & \frac{x^*}{\sqrt{|x|^2+|y|^2}} & \\ & & & I_{d-j-1} \end{pmatrix},$$

$$Z_d(\theta) = \sum_{j=0}^{d-1} \mathrm{e}^{\mathrm{i}(1-\mathrm{sgn}(d-1-j))\theta} |j\rangle\langle j|,$$

$$C_2[R_d] = \begin{pmatrix} I_{d^2-d} & \\ & R_d \end{pmatrix}$$

$$R_d \longrightarrow |d-1\rangle|0\rangle, \cdots, |d-1\rangle|d-1\rangle$$

$$I_{d^2-d} \longrightarrow |0\rangle|0\rangle, \cdots, |d-2\rangle|d-1\rangle$$

$$U_d(\boldsymbol{\alpha}) : \sum_{l=0}^{d-1} \alpha_l |l\rangle \mapsto |d-1\rangle, \; \boldsymbol{\alpha} := (\alpha_0, \alpha_1, \ldots, \alpha_{d-1}).$$

$$X_j(x,y) = \begin{pmatrix} I_{j-1} & & & \\ & \frac{x}{\sqrt{|x|^2+|y|^2}} & \frac{-y}{\sqrt{|x|^2+|y|^2}} & \\ & \frac{y^*}{\sqrt{|x|^2+|y|^2}} & \frac{x^*}{\sqrt{|x|^2+|y|^2}} & \\ & & & I_{d-j-1} \end{pmatrix},$$

$$Z_d(\theta) = \sum_{j=0}^{d-1} e^{i(1-\mathrm{sgn}(d-1-j))\theta} |j\rangle\langle j|,$$

$$C_2[R_d] = \begin{pmatrix} I_{d^2-d} & \\ & R_d \end{pmatrix}$$

$$U_d = X_d^{(d-1)}(a_{d-1}, b_{d-1}) \cdots X_d^{(1)}(a_1, b_1)$$

$$a_l := \alpha_l, \; b_l := \sqrt{\sum_{l=0}^{l-1} \alpha_i^2}$$

$$U \in SU(d^n)$$

$$U = \sum_{j=1}^{N} e^{i\lambda_j} |E_j\rangle\langle E_j| = \prod_{j=1}^{N} \Upsilon_j$$

$$\Upsilon_j = \sum_{s=1}^{N} e^{i(1-|\mathrm{sgn}(j-s)|)\lambda_s} |E_s\rangle\langle E_s|.$$

$$\Upsilon_j = U_{j,N}^{-1} Z_{j,N} U_{j,N}.$$

$$U_{j,N}(\alpha_0, \cdots, \alpha_{N-1}) : |E_j\rangle \mapsto |N-1\rangle$$

$$Z_{j,N} = \sum_{s=0}^{N-1} e^{i(1-|\mathrm{sgn}(s-N+1)|)\lambda_j} |s\rangle\langle s|.$$

$U_d(\alpha) : \sum_{l=}^{d-1}$

$U_d = X_d^{(d-1)}(a_{d-1}$

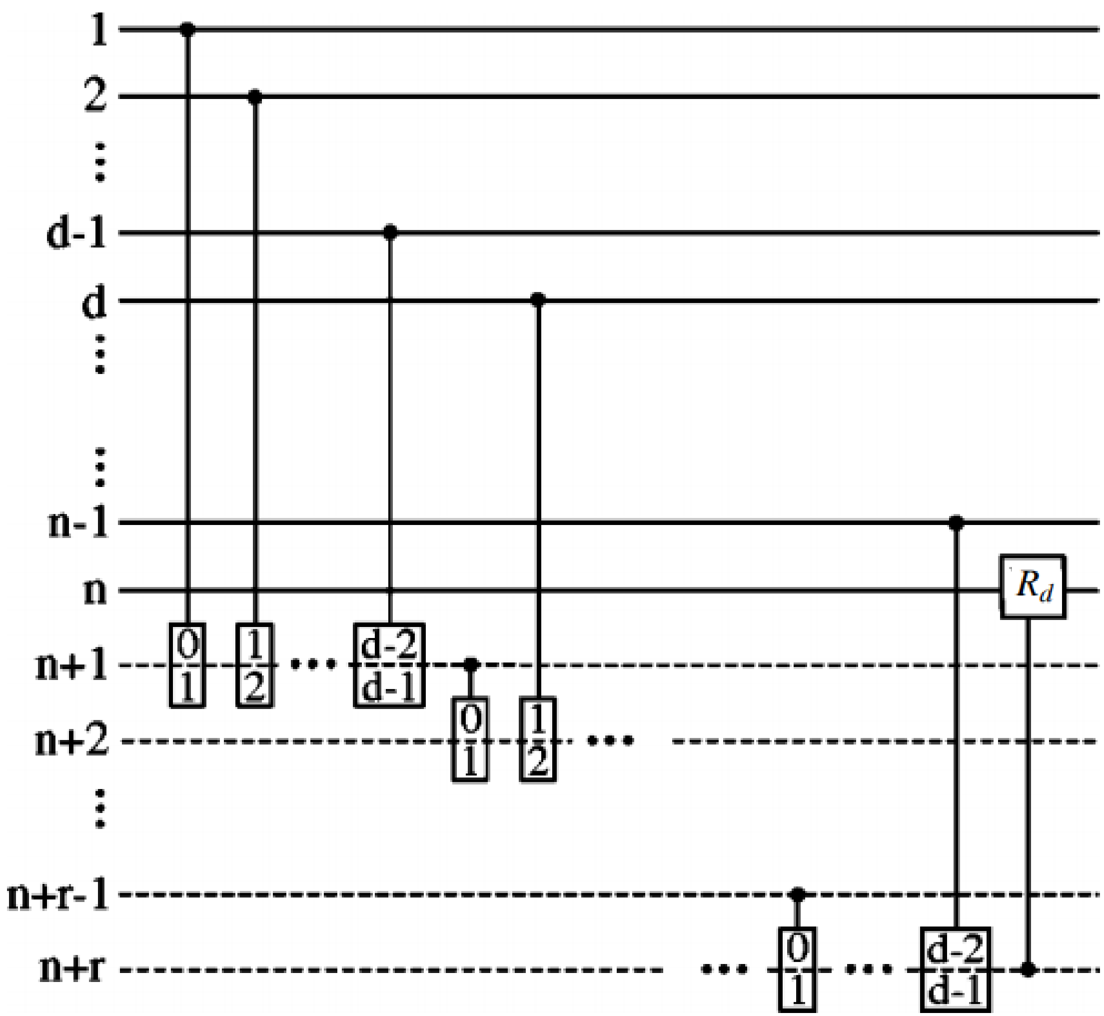
$a_l := \alpha_l,\ b_l := \sqrt{}$

$U \in SU(d^n)$

$U = \sum_{j=1}^{N} e^{i}$

$\Upsilon_j = \sum_{s=1}^{N} e^{i(1-}$

$\Upsilon_j = U_{j,N}^{-1} Z_{j,}$



$\frac{-y}{\sqrt{|x|^2+|y|^2}}$ $\frac{x^*}{\sqrt{|x|^2+|y|^2}}$ I_{d-j-1}

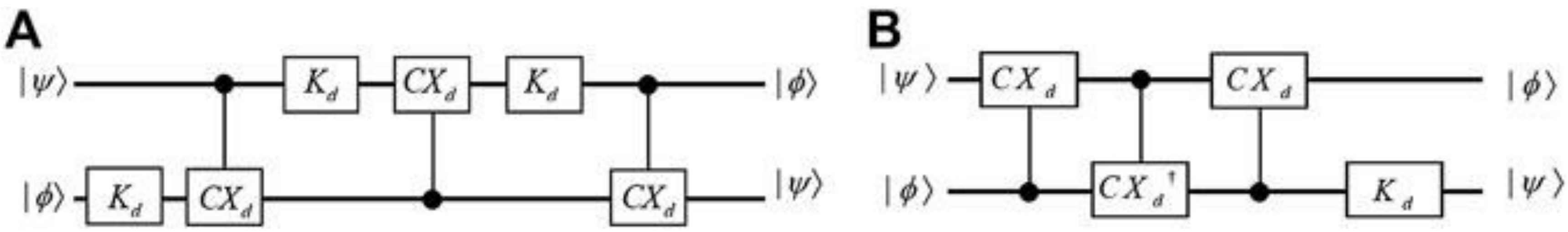
$\mathrm{sgn}(d-1-j))\theta|j\rangle\langle j|,$

$R_d)$

$\rangle \mapsto |N-1\rangle$

$+1)|)\lambda_j|s\rangle\langle s|.$

SWAP Gate



$$CX_d \left|x\right\rangle \left|y\right\rangle = \left|x\right\rangle \left|x+y\right\rangle$$

$$CX_d \neq CX_d^{\dagger}.$$

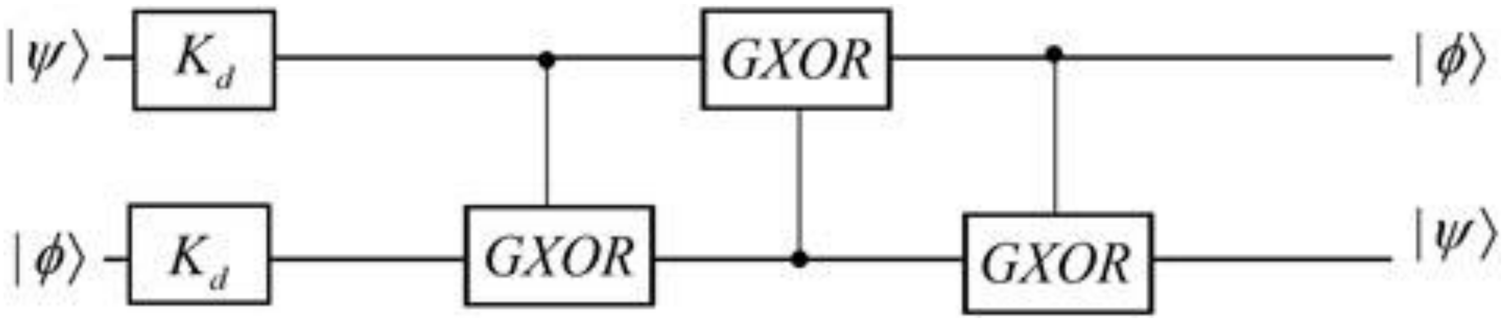
$$CX_d^{\dagger} \left|x\right\rangle \left|y\right\rangle = \left|x\right\rangle \left|y-x\right\rangle$$

$$K_d \left|x\right\rangle = \left|d-x\right\rangle = \left|-x\right\rangle,$$

Fujii, K. Exchange gate on the qudit space and fock space. J Opt B Quantum Semiclassical Opt (2003). 5:S613–S618.

Paz-Silva, GA, Rebić, S, Twamley, J, and Duty, T. Perfect mirror transport protocol with higher dimensional quantum chains. Phys Rev Lett (2009). 102:020503.

SWAP Gate



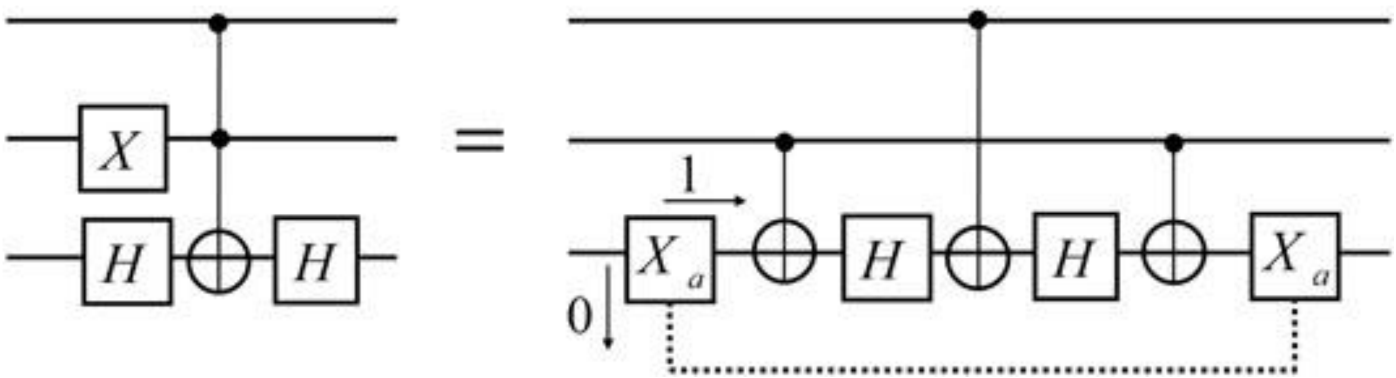
$$GXOR\,|x\rangle\,|y\rangle = |x\rangle\,|x-y\rangle\,.$$

$$K_d\,|x\rangle = |d-x\rangle = |-x\rangle\,,$$

Wang, X. Continuous-variable and hybrid quantum gates. J Phys Math Gen (2001). 34:9577–84.

Alber, G, Delgado, A, Gisin, N, and Jex, I. Efficient bipartite quantum state purification in arbitrary dimensional Hilbert spaces. J Phys Math Gen (2001). 34:8821–33.

Simplified qubit Toffoli gate with a qudit



$$X_a |0\rangle = |2\rangle$$
$$X_a |2\rangle = |0\rangle$$
$$X_a |1\rangle = |1\rangle.$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$
$$H|2\rangle = |2\rangle$$

$$X_m |0\rangle = |m\rangle, X_m |m\rangle = |0\rangle, X_m |y\rangle = |y\rangle, \text{for } y \neq m, 0.$$

Спасибо за внимание!