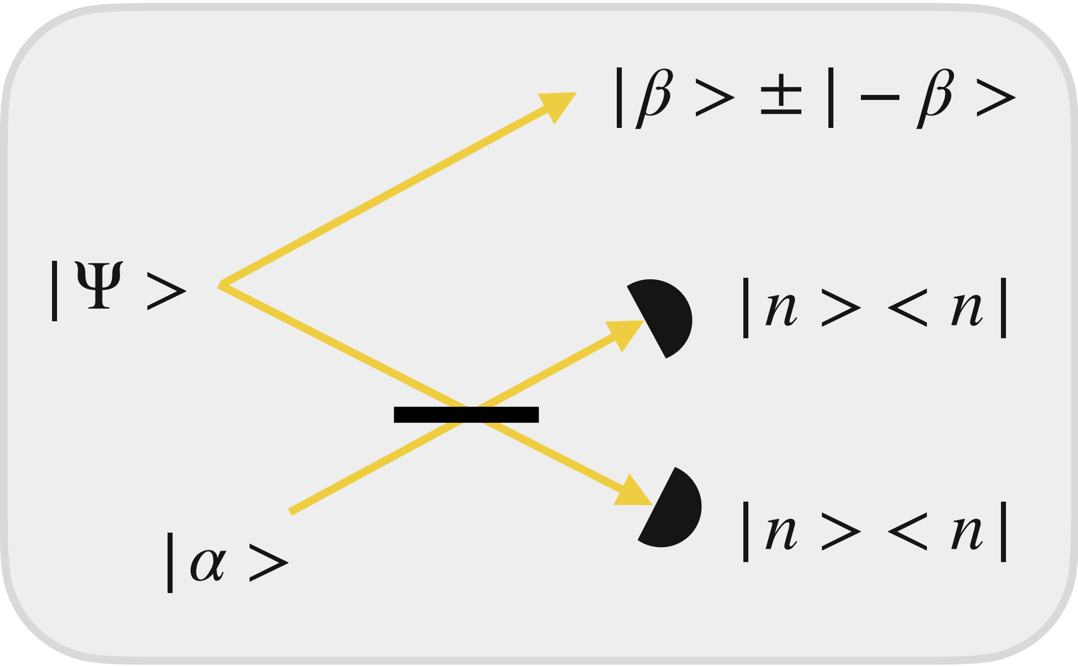
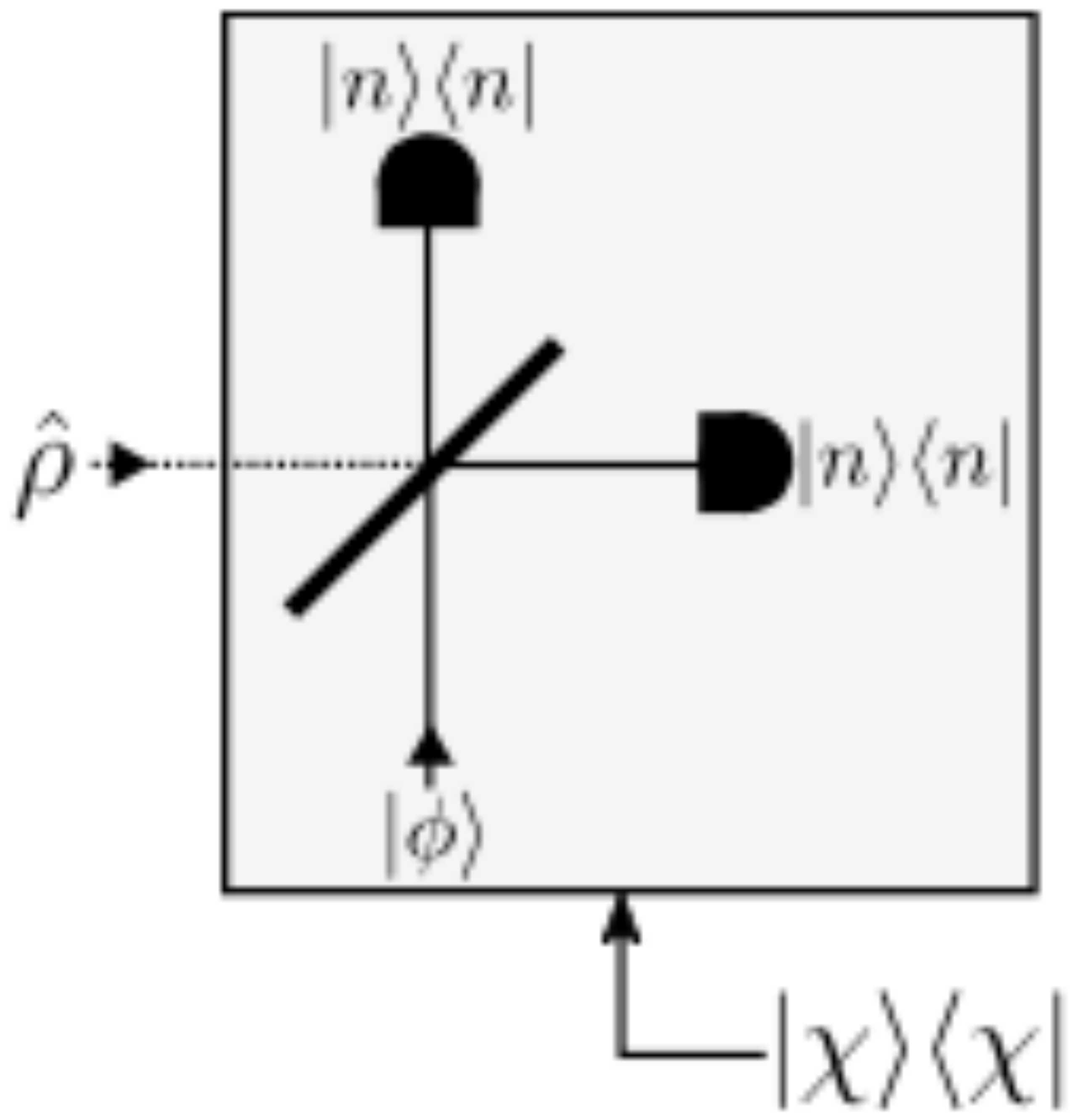
Создание кота Шредингера с помощью «детектора четности»

G.S. Thekkadath, B.A. Bell, I.A. Walmsley, and A.I. Lvovsky.
Engineering Schrödinger cat states with a photonic even-parity detector.
Quantum 4, 239 (2020)

А.С. Лосев - Наш семинар - 30.09.2021



Детектор четности



$$|\phi\rangle = \sum_{m=0}^{\infty} c_m |m\rangle \quad (1)$$

$$\sum_m |c_m|^2 = 1$$

$$\mathrm{pr}(n,n) = \left\langle n,n \left| \hat{U} \left[\hat{\rho} \otimes |\phi\rangle\langle\phi|\right] \hat{U}^\dagger \right| n,n \right\rangle \quad (2)$$

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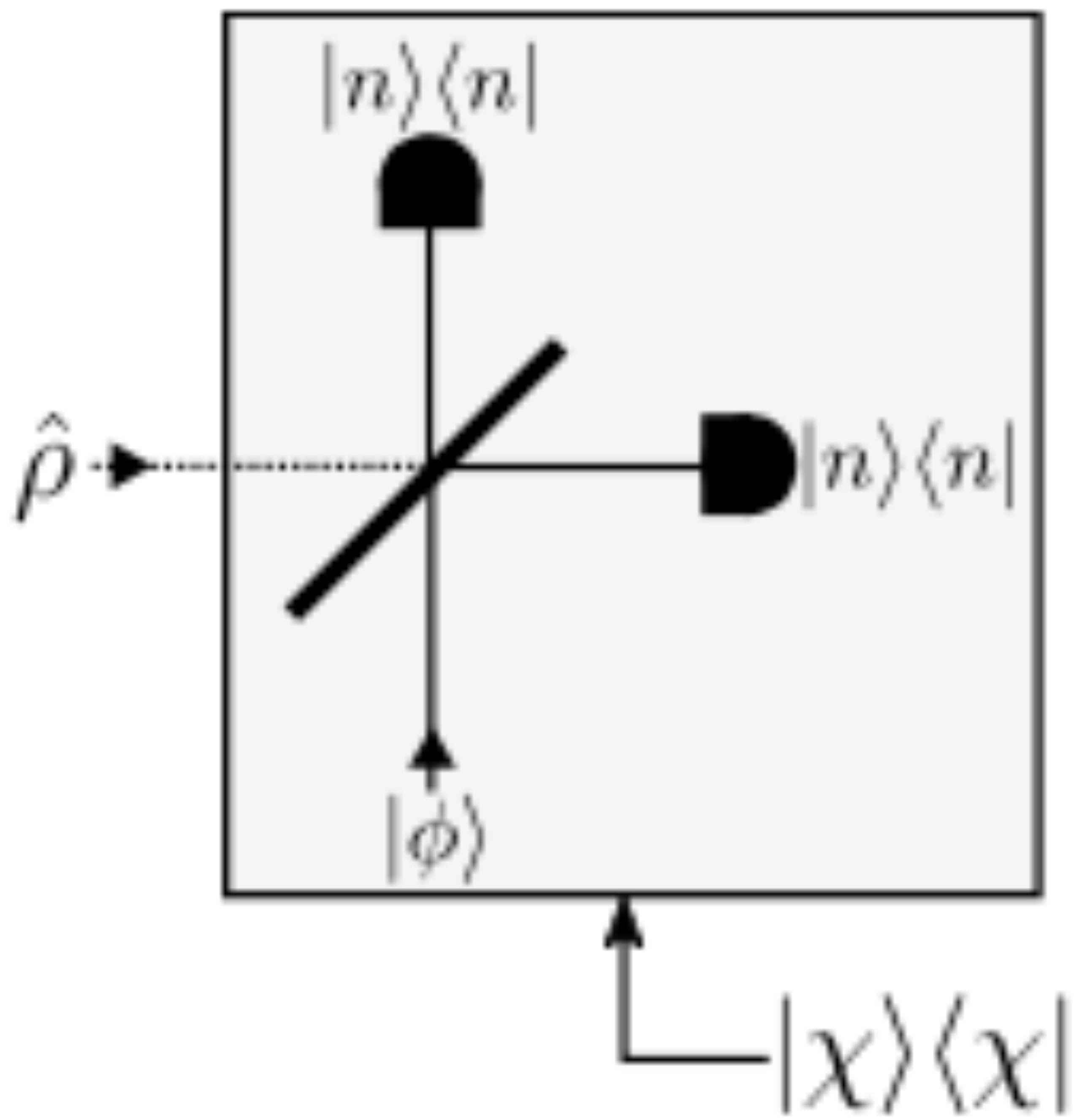

$$\mathrm{pr}(n,n) = \left\langle n,n \left| \hat{U} \left[\hat{\rho} \otimes |\phi\rangle \langle\phi|\right] \hat{U}^\dagger \right| n,n \right\rangle \quad (2)$$

$$\mathrm{pr}(n,n) = \langle\chi|\hat{\rho}|\chi\rangle \quad (2a)$$

$$|\chi\rangle = \langle\phi|\hat{U}^\dagger|n,n\rangle = \sum_{j=0}^{2n} c^*_{2n-j} A_{j,n} |j\rangle \quad (3)$$

$$A_{j,n} = \langle j, 2n-j|\hat{U}|n,n\rangle = \begin{cases} \left(\frac{i}{2}\right)^n \frac{\sqrt{(2n-j)!(j)!}}{(j/2)!(n-j/2)!} & \text{for even } j \\ 0 & \text{for odd } j \end{cases} \quad (4)$$

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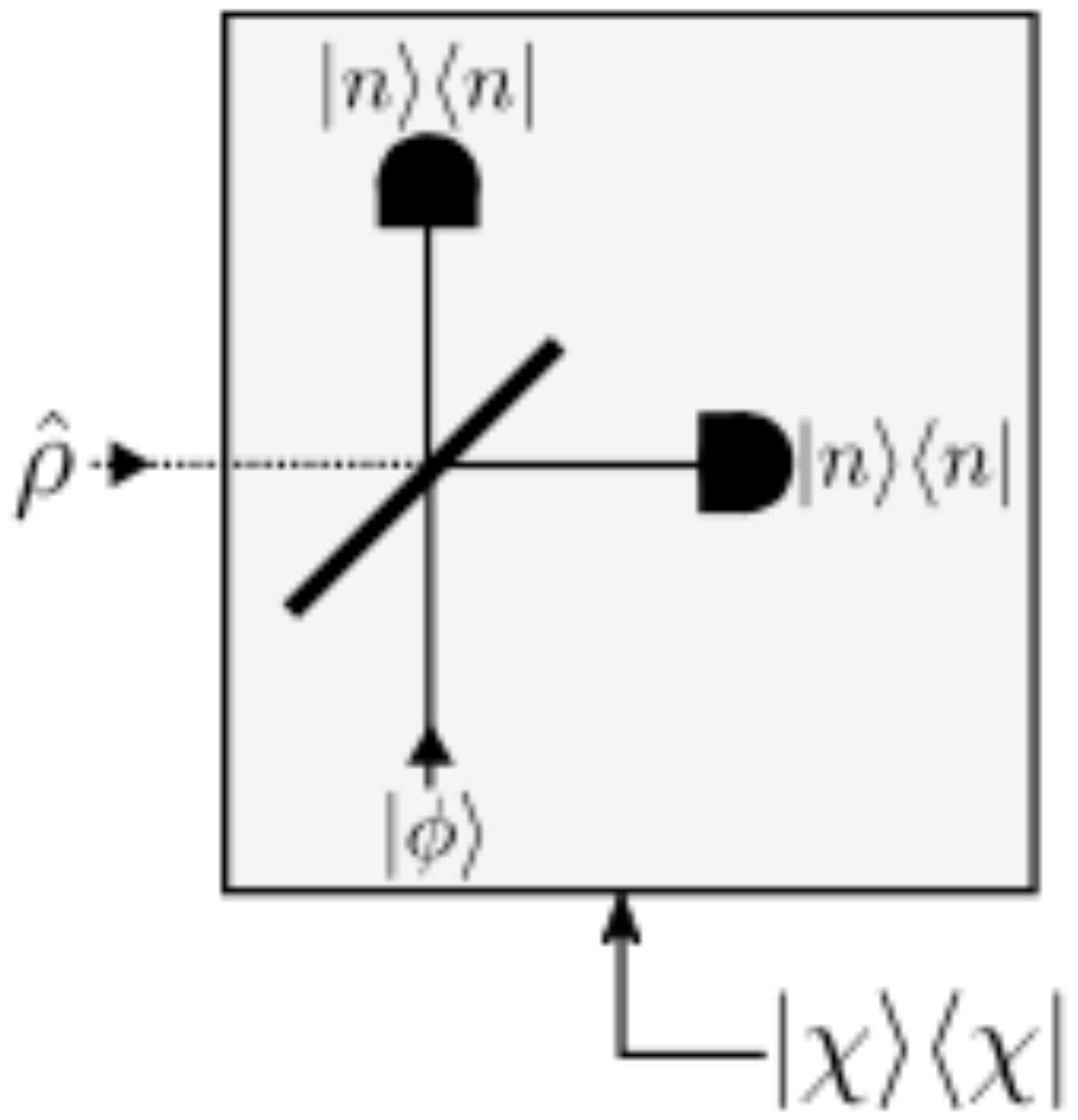

$\hat{U}\,|n, n\rangle$ - состояние Холланда-Барнета

[1] R.A. Campos, B.E.A. Saleh, and M.C. Teich. Phys. Rev. A **40**, 1371 (1989).

[2] M.J. Holland and K. Burnett. Phys. Rev. Lett. **71**, 1355 (1993).

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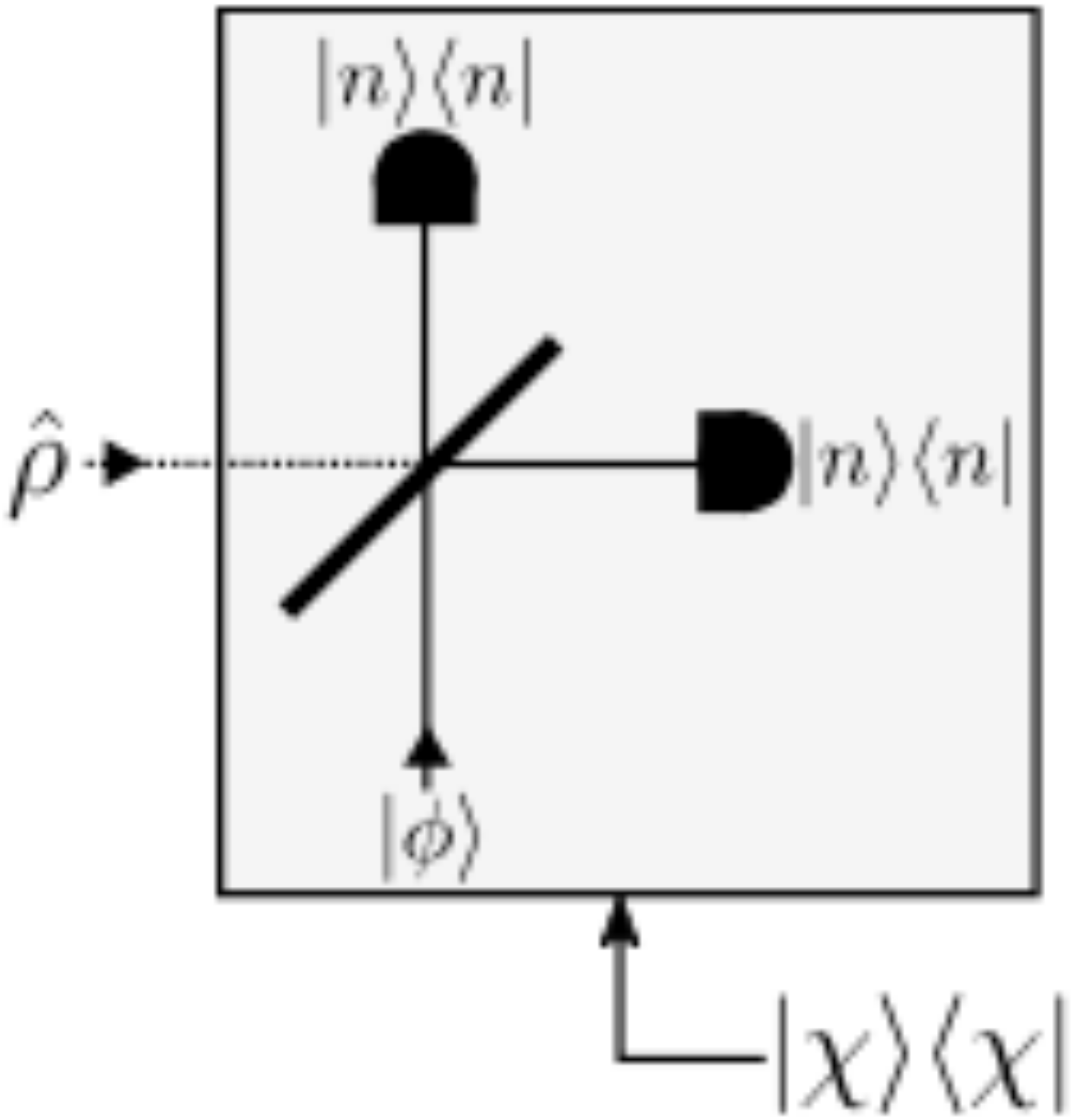

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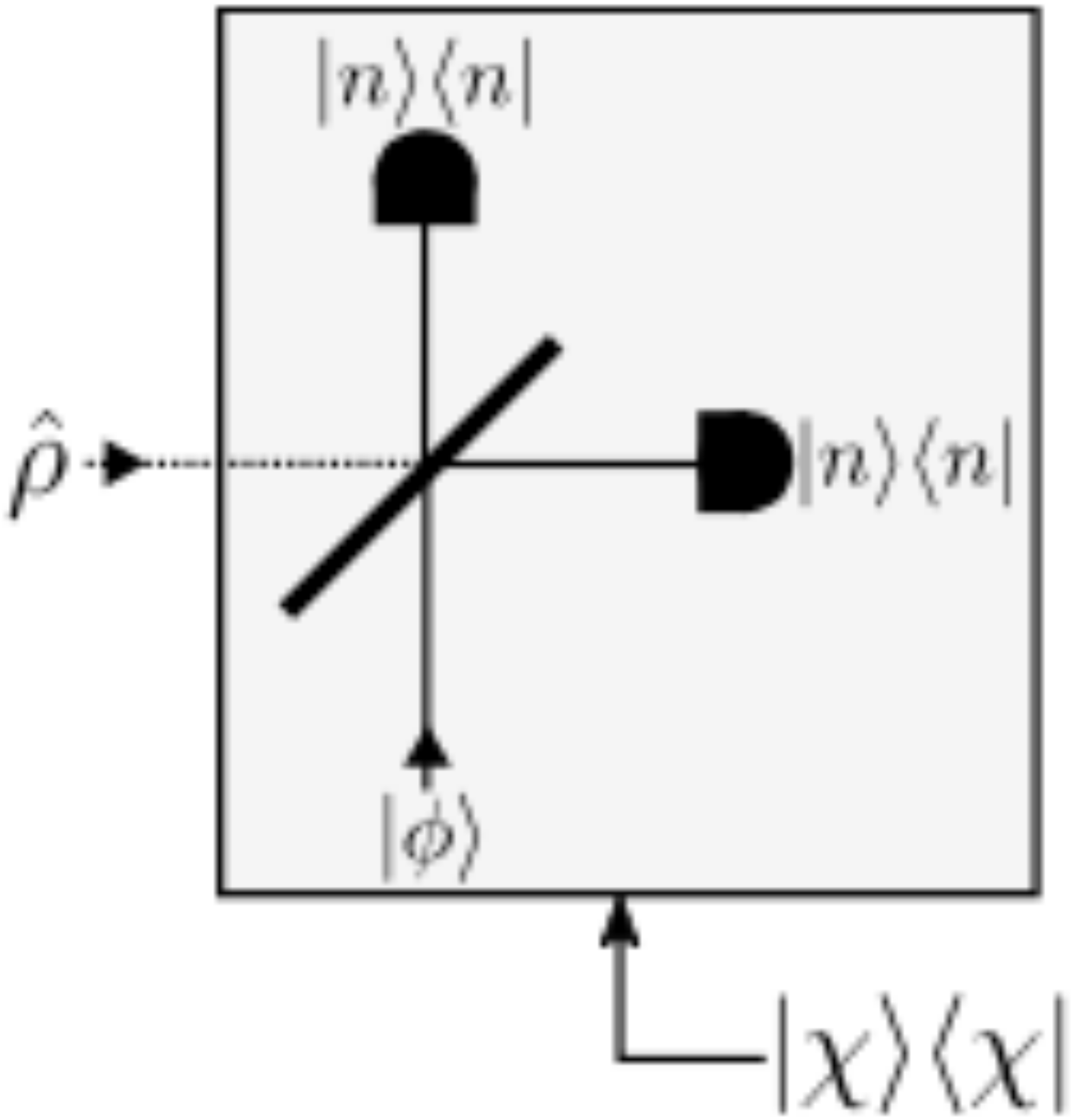
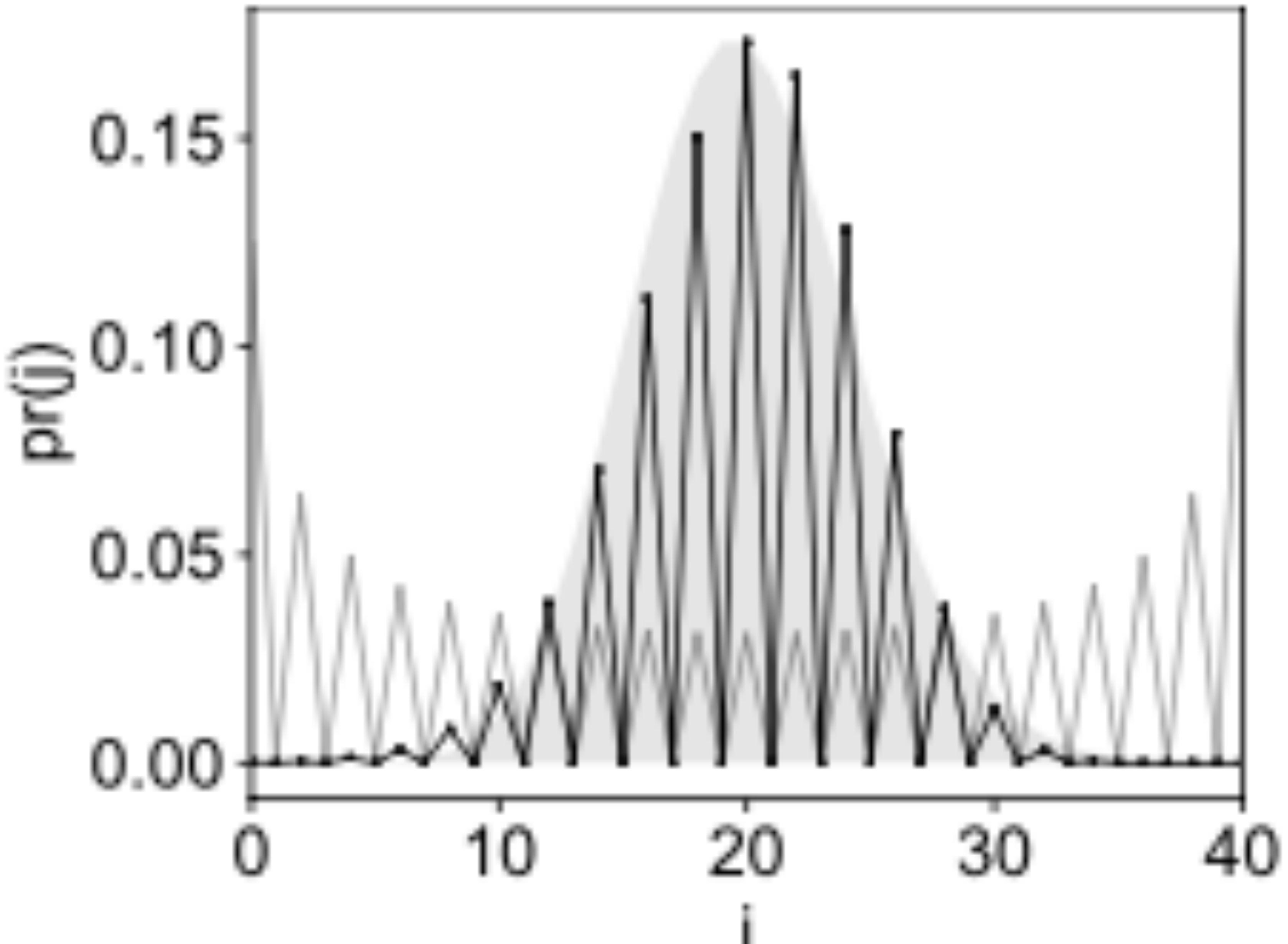
$$A_{j,n} \approx \frac{i^n}{\sqrt{\pi}} \frac{1}{[(j/2)(n-j/2)]^{1/4}} \approx \sqrt{\frac{2}{n\pi}} i^n \left(1 + \frac{1}{4n^2}(j-n)^2\right) + \mathcal{O}(j-n)^4 \quad (5)$$

Детектор четности для когерентного состояния





$|\phi> = |\beta>$

$|\beta|^2 = 20$

при этом $n = 20$

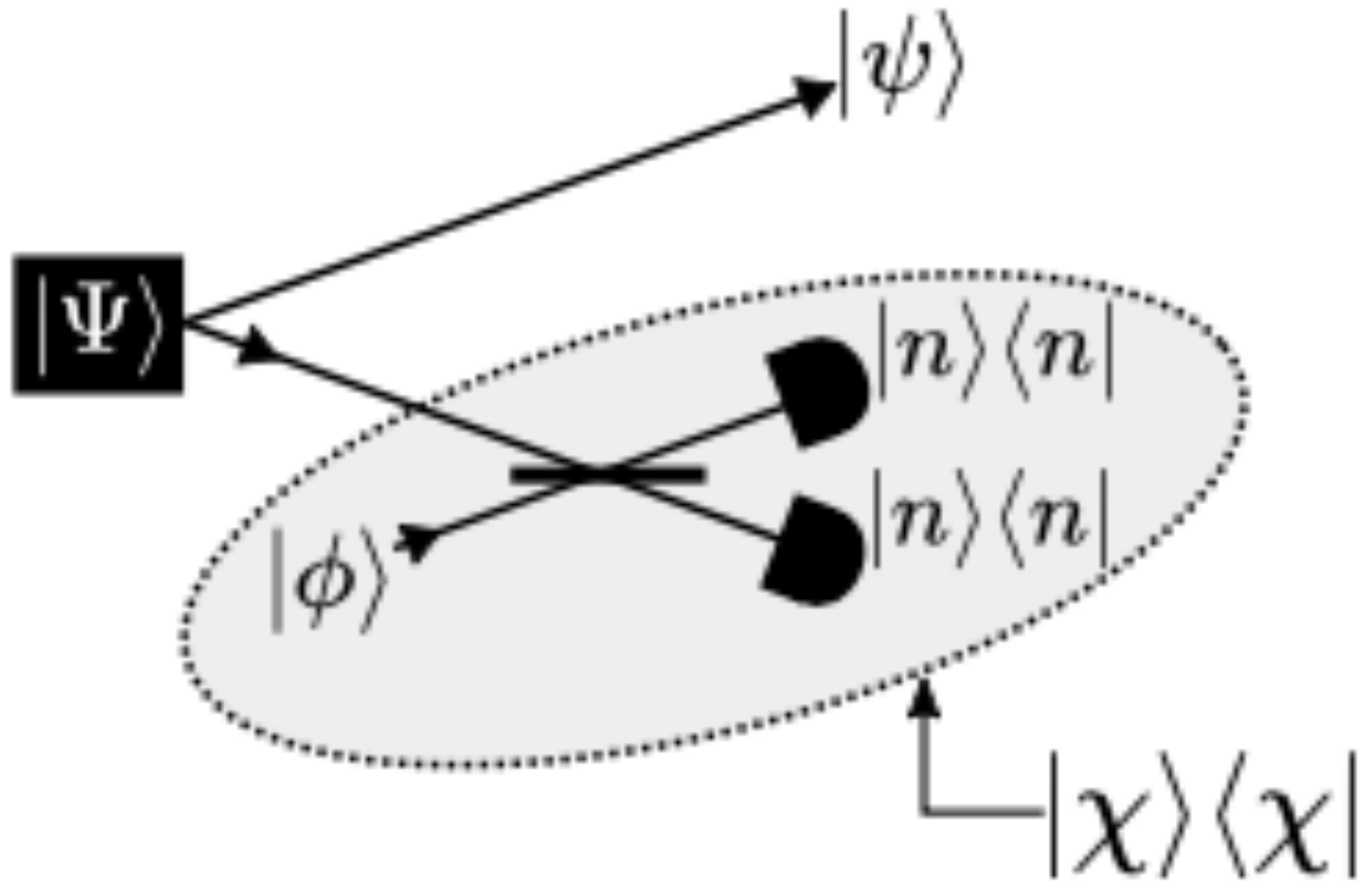
(то есть $c_j \approx c_{2n-j}$)

$$|\chi\rangle = \langle\phi|\hat{U}^\dagger|n,n\rangle = \sum_{j=0}^{2n} c^*_{2n-j} A_{j,n}\,|j\rangle$$

$$|\chi\rangle \approx |\beta\rangle + |-\beta\rangle$$

$$\mathrm{pr}(n,n) = \langle\chi|\hat{\rho}|\chi\rangle$$

Формирование состояний с помощью детектора четности



$$|\Psi\rangle = \sqrt{1-\lambda^2}\sum_{k=0}^{\infty}\lambda^j\,|k,k\rangle\,, \qquad (6)$$

$\lambda = th(r)$, r — параметр сжатия

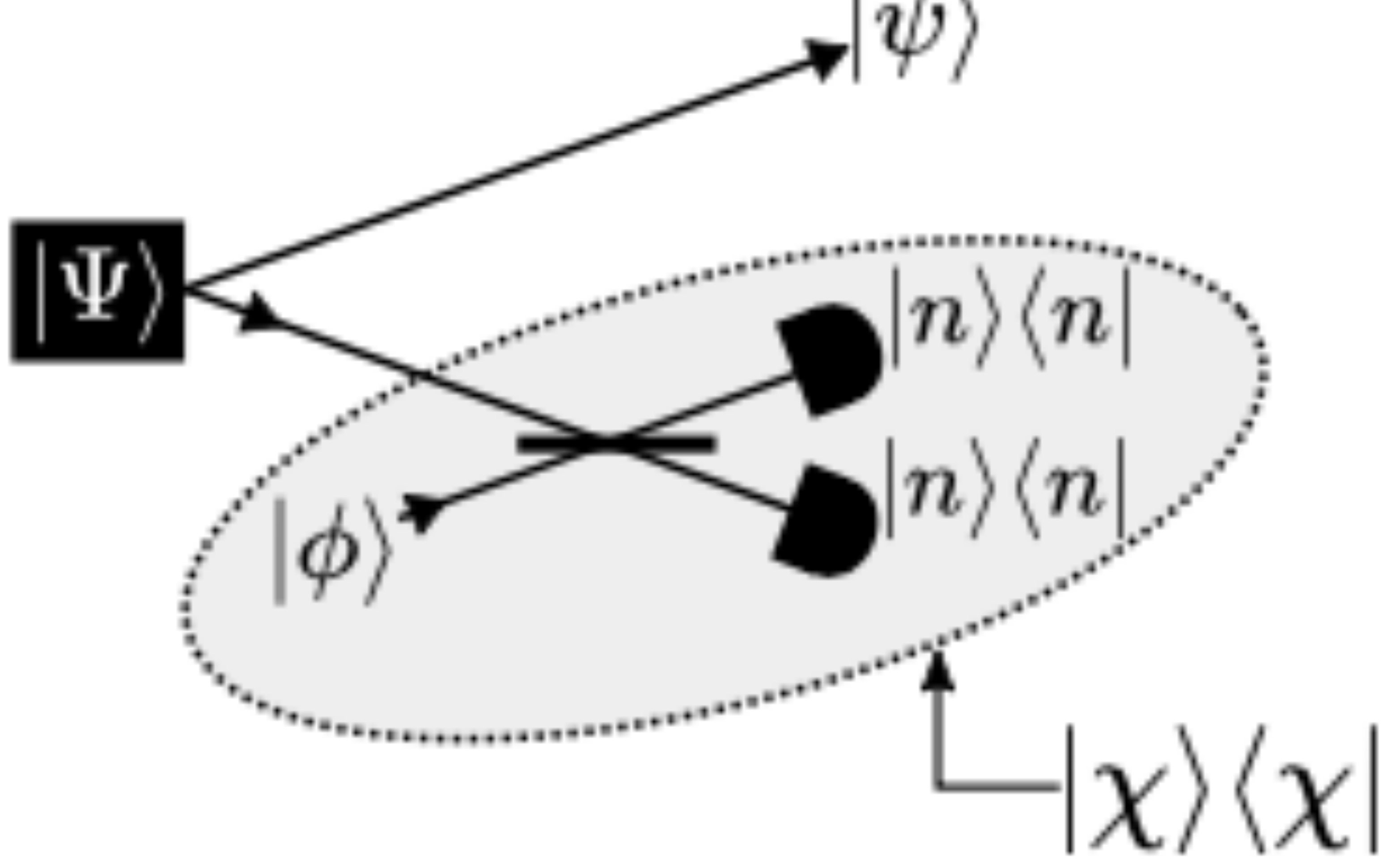
$$|\psi\rangle = \langle\chi|\Psi\rangle = \sqrt{1-\lambda^2}\sum_{j=0}^{2n} c_{2n-j}\lambda^j A^*_{j,n}\,|j\rangle \qquad (7)$$

$$|\chi\rangle = \langle\phi|\hat{U}^\dagger|n,n\rangle = \sum_{j=0}^{2n} c^*_{2n-j} A_{j,n}\,|j\rangle$$

Двухкомпонентный кот

$$|\mathrm{cat}_\beta\rangle = \mathcal{N}\left(|\beta\rangle + |-\beta\rangle\right)$$

$$\mathcal{N} = \left[2(1+e^{-2|\beta|^2})\right]^{-1/2}$$



$$|\phi\rangle = |\alpha\rangle \qquad \alpha = \beta\lambda$$

$$\begin{aligned} |\psi_{\mathrm{cat}}\rangle &= \mathcal{N}' \sum_{j=0}^{2n} \frac{\alpha^{2n-j}\lambda^j}{\sqrt{(2n-j)!}} A^*_{j,n}\,|j\rangle \\ &= \mathcal{N}'\lambda^{2n} \sum_{j=0}^{2n} \frac{\beta^{2n-j}}{\sqrt{(2n-j)!}} A^*_{j,n}\,|j\rangle \end{aligned} \qquad (8)$$

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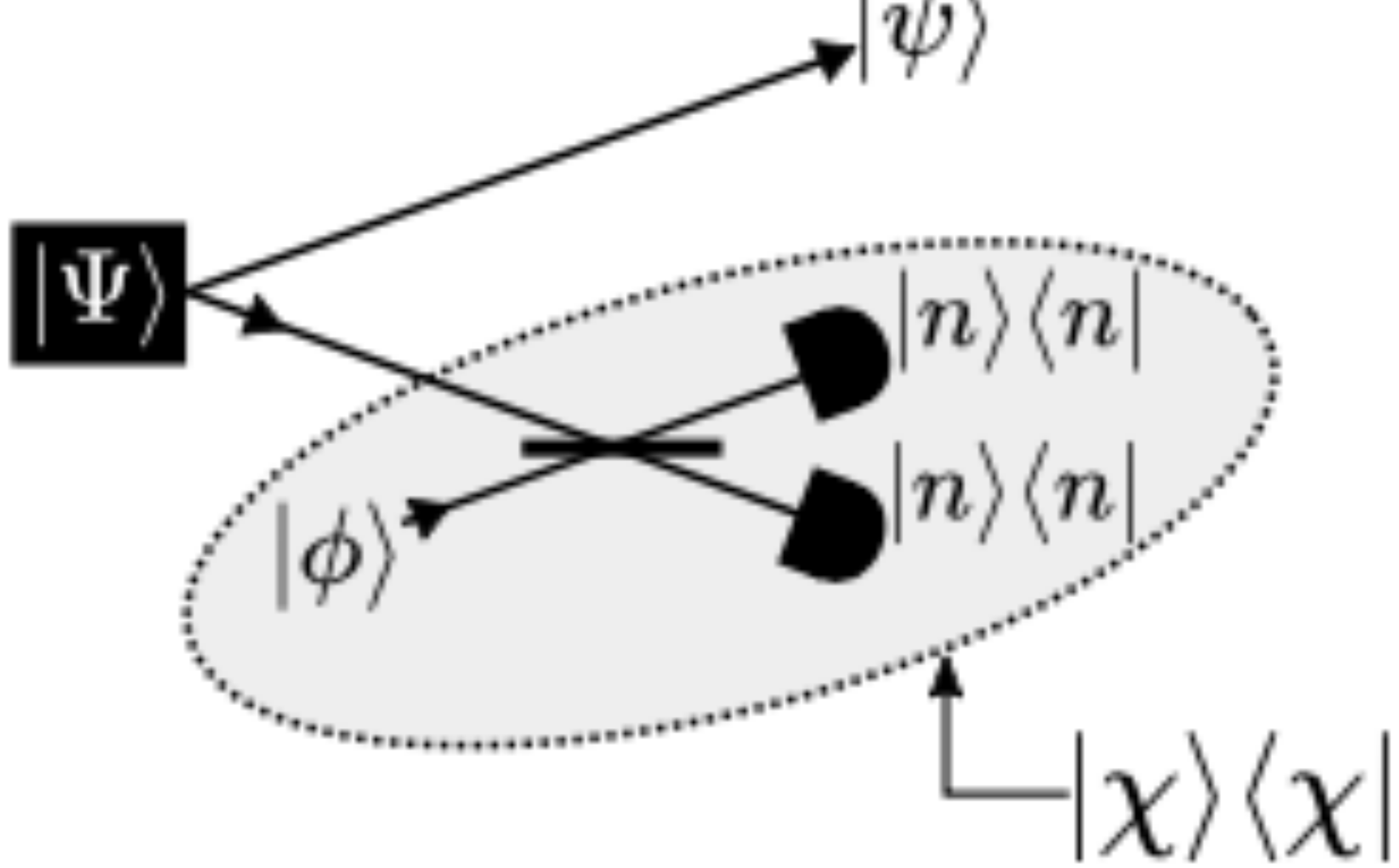

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$$\mathcal{F} = |\langle\psi_{\mathrm{cat}}|\mathrm{cat}_\beta\rangle|^2,$$

$$n \to |\beta|^2$$

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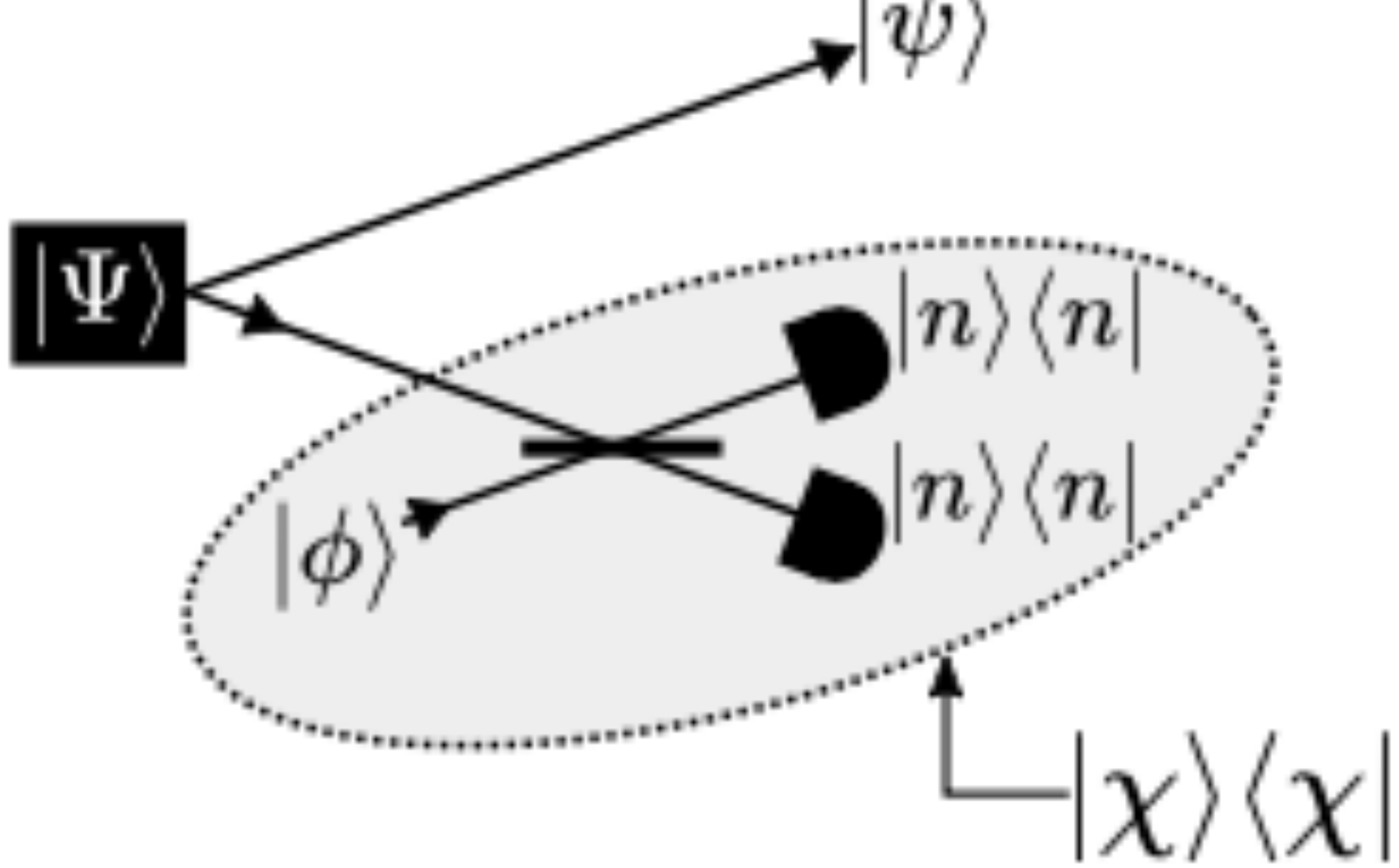

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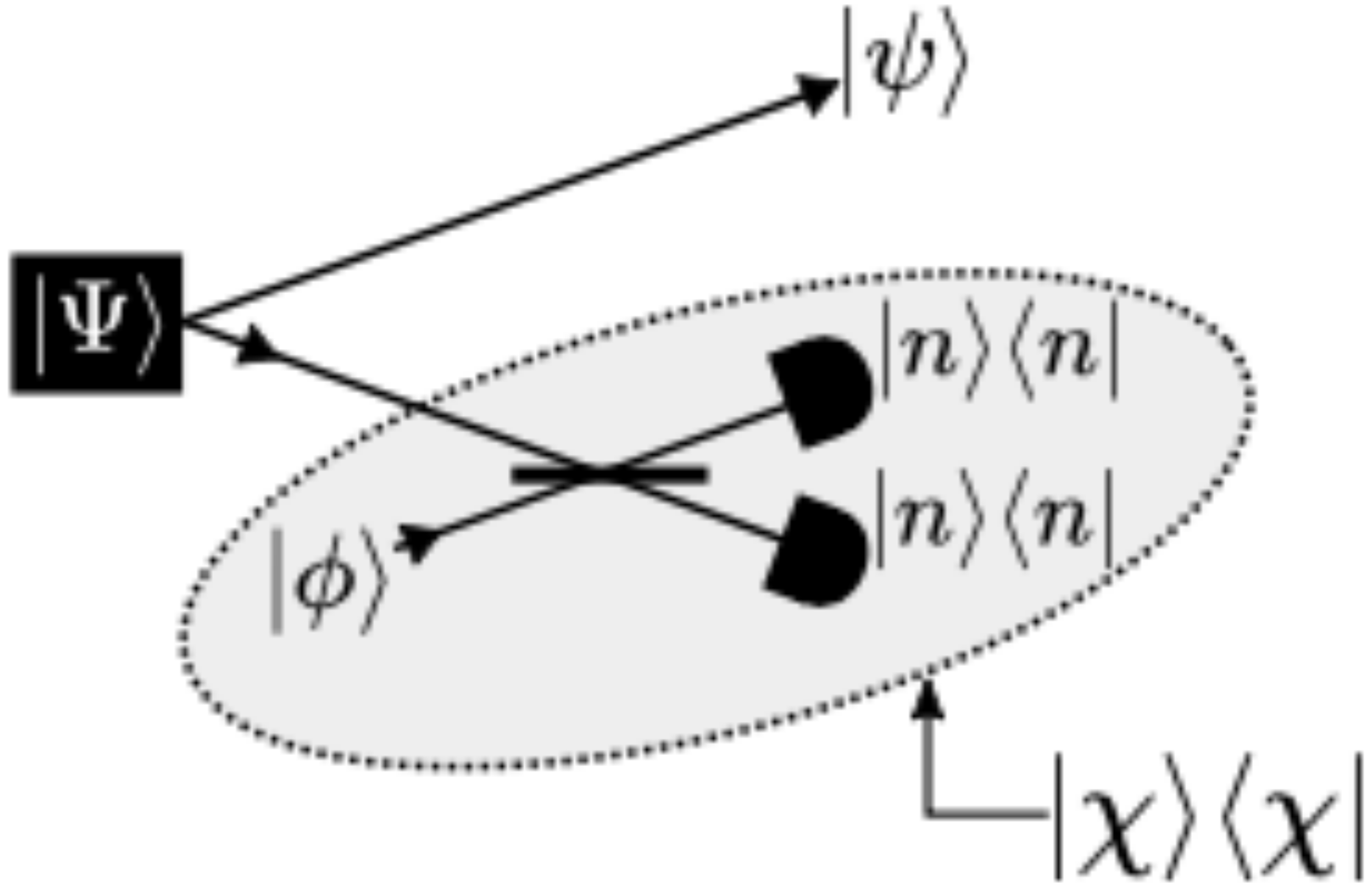
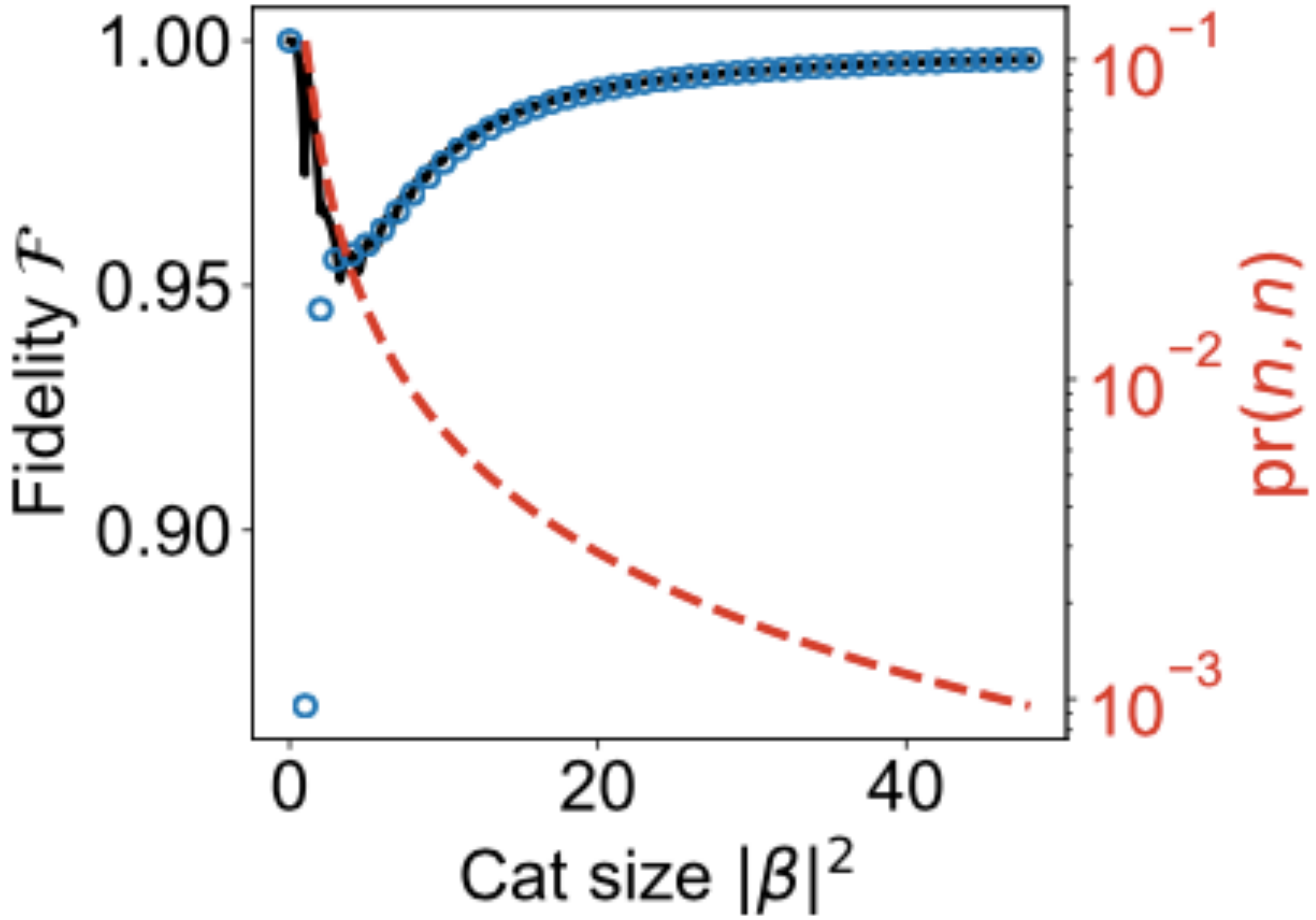
$$\mathcal{F} = |\langle\psi_{\mathrm{cat}}|\mathrm{cat}_\beta\rangle|^2 \qquad n \to |\beta|^2$$

$$|\beta|^2 = n: \quad \mathcal{F} = \frac{2^{2n+1}e^{-n}}{(1+e^{-2n})}\left(\sum_{k=0}^{n}\binom{n}{k}^2\frac{(2k)!}{n^{2k}}\right)^{-1} \quad (9)$$

Двухкомпонентный кот





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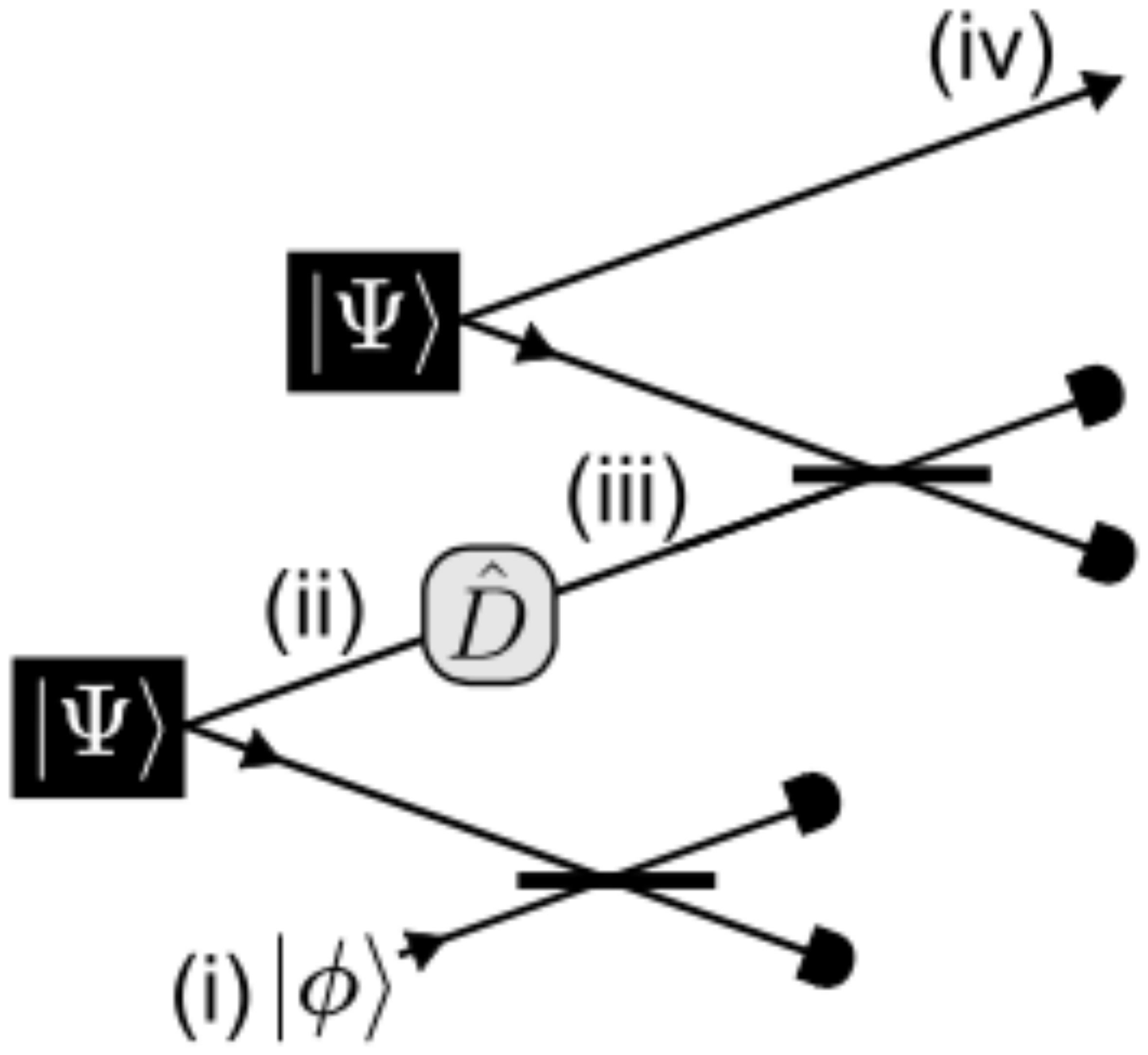
Четырёхголовый кот



$\lambda = 1$

$$|\psi_{\text{iii}}\rangle = \hat{D}(i\beta)\left(|\beta\rangle + |-\beta\rangle\right) = e^{i2|\beta|^2}\,|\beta + i\beta\rangle + e^{-i2|\beta|^2}\,|-\beta + i\beta\rangle \quad (10)$$

$$|\psi_{\text{iv}}\rangle = |\beta - i\beta\rangle + |-\beta + i\beta\rangle + e^{-i2|\beta|^2}\left(|\beta + i\beta\rangle + |-\beta - i\beta\rangle\right) \quad (11)$$

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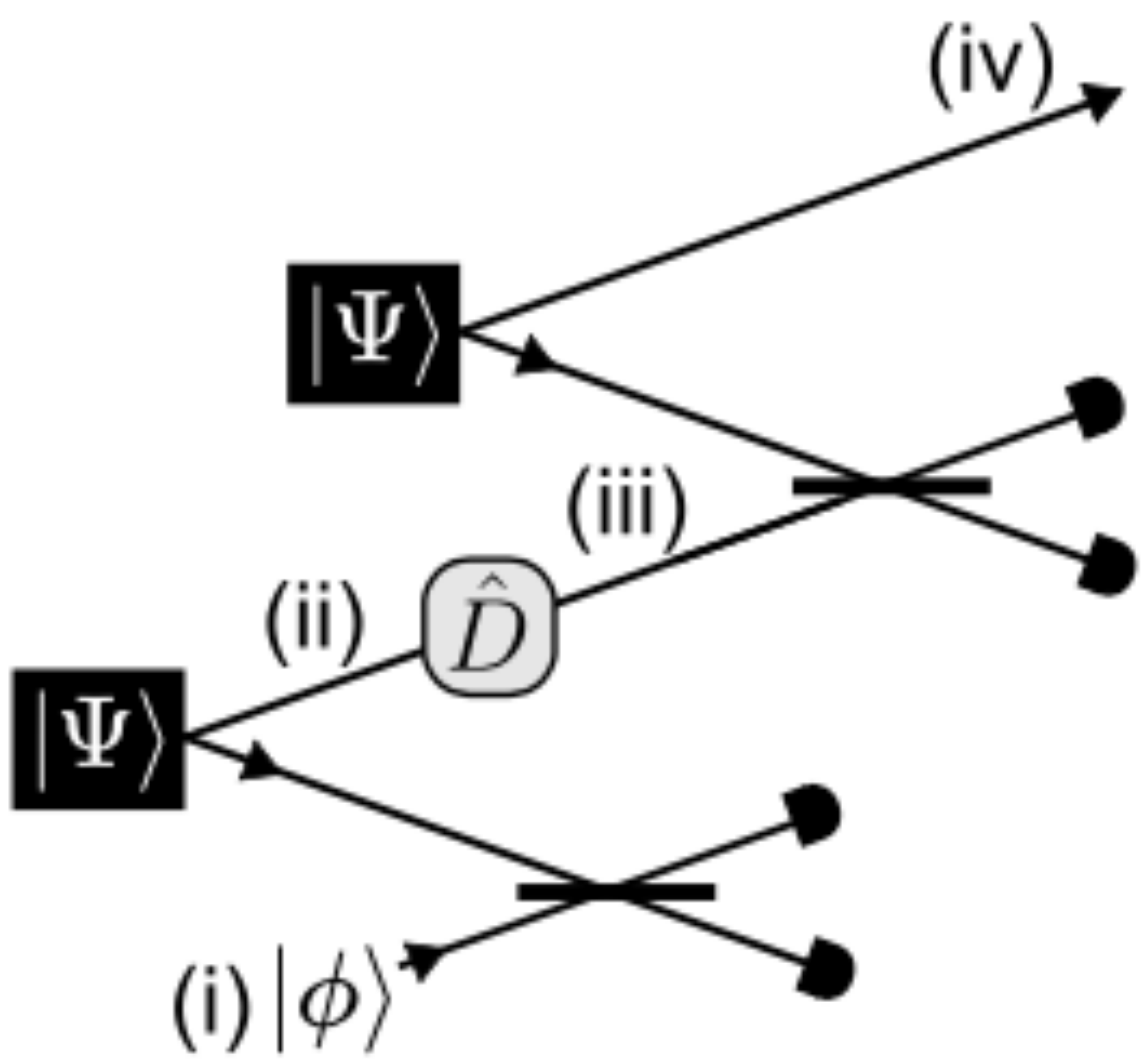

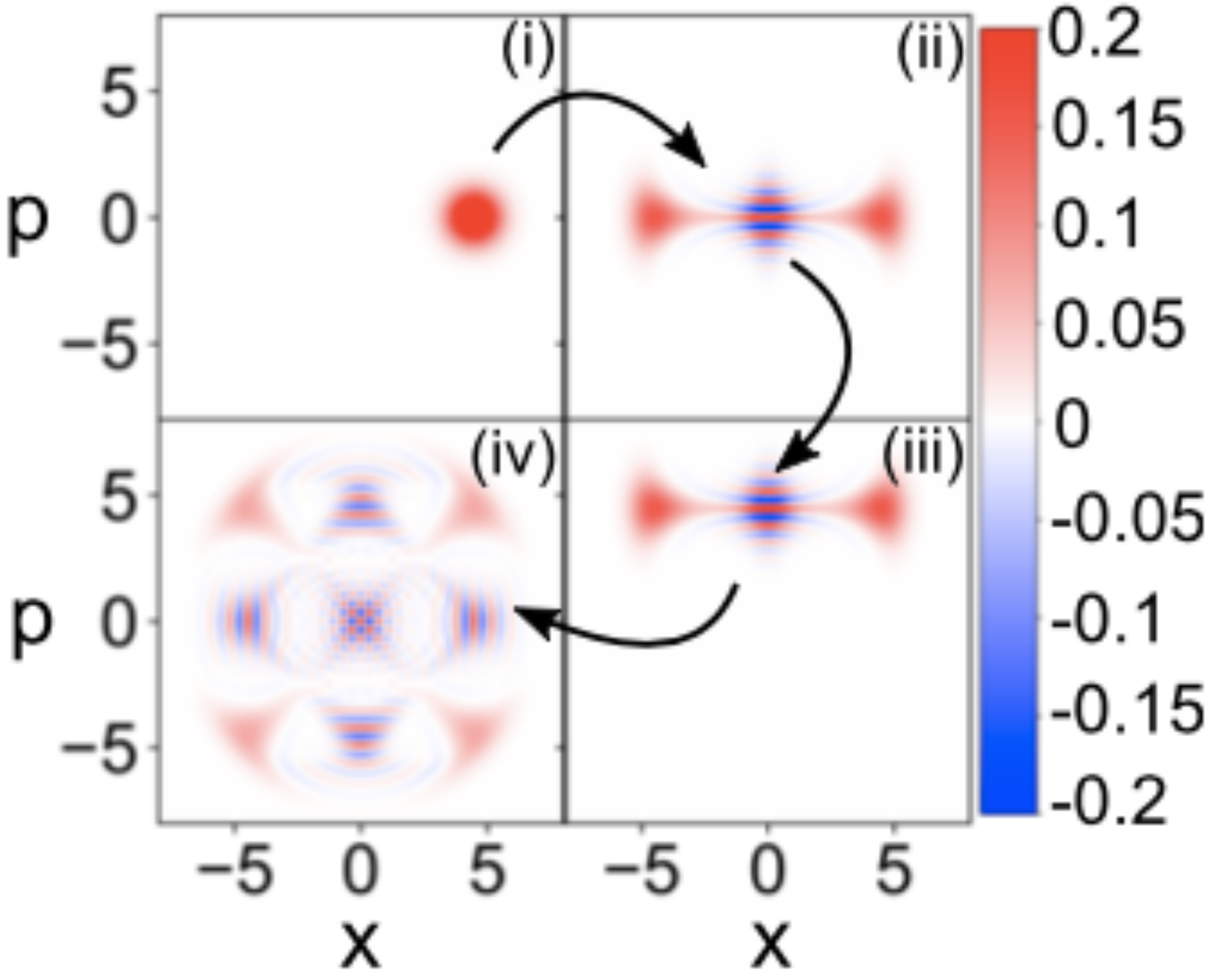

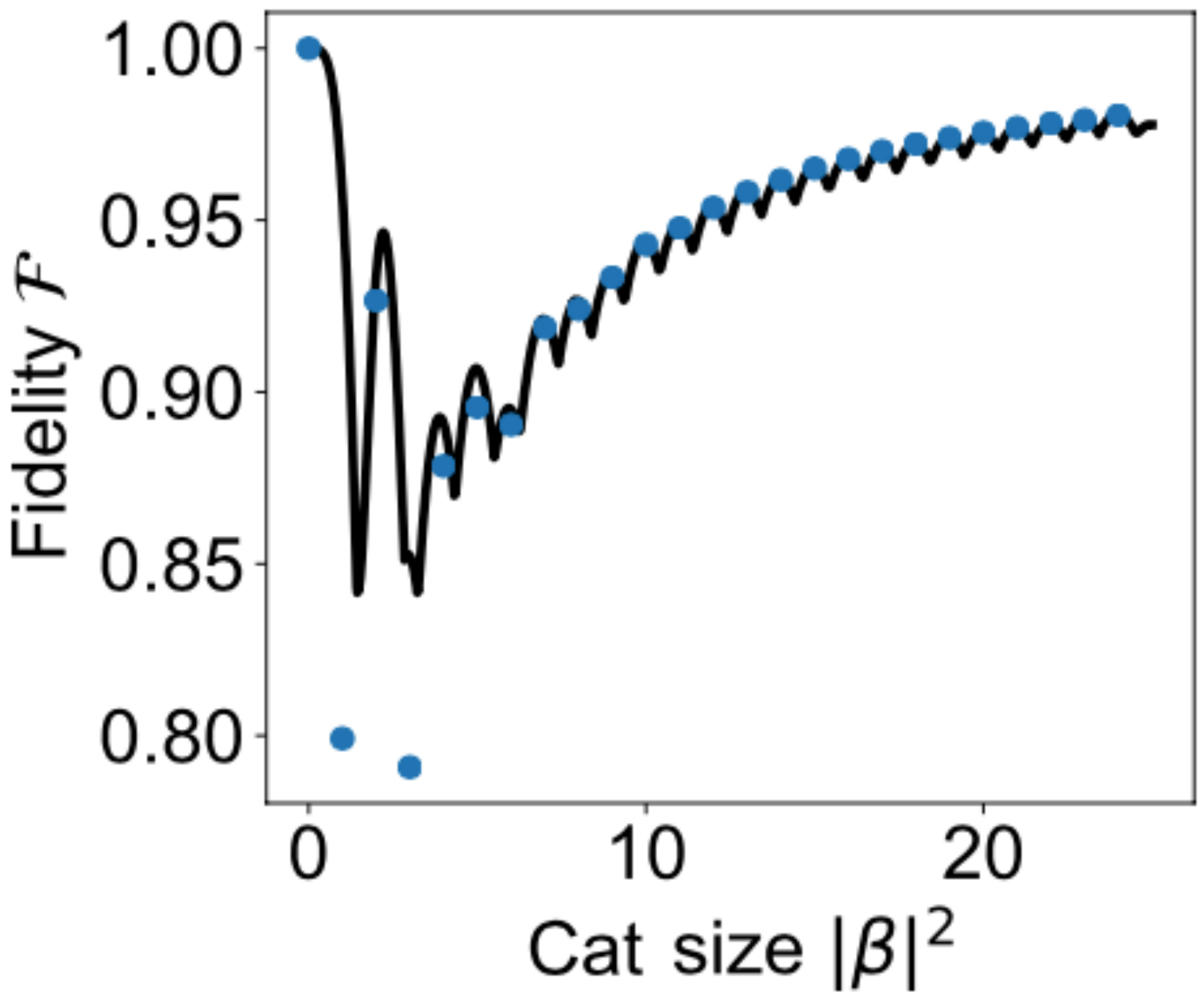

$$
\begin{aligned}
|\psi_{\mathrm{iv}}\rangle &= |\beta - i\beta\rangle + |-\beta + i\beta\rangle \\
&\quad + e^{-i2|\beta|^2}\left(|\beta + i\beta\rangle + |-\beta - i\beta\rangle\right)
\end{aligned}
\tag{11}
$$