Физический смысл радиального индекса Лагерр-Гауссовых мод

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План доклада

- Лагерр-Гауссовы моды
- Оператор ОУМ
- Оператор числа квазичастиц в поперечной плоскости
- Оператор радиального индекса Лагерр-Гауссовых мод
- Связь Лагерр-Гауссовых мод и мод Цернике
- Стоячая волна круглой мембраны

Лагерр-Гауссовы моды

Параксиальное приближение: $\frac{\partial^2 u}{\partial z^2} \ll k\frac{\partial u}{\partial z} \ll ku\alpha.$

$$\nabla_\perp^2 E - 2ik\frac{\partial}{\partial z}E = 0, \tag{1}$$

$$LG_{pl}(\rho, \phi, z) = \frac{N_{pl}}{\omega_z}\left(\frac{\rho\sqrt{2}}{\omega_z}\right)^{|l|} L_p^{|l|}\left(\frac{2\rho^2}{\omega_z^2}\right)\exp\left(\left(i\frac{z}{z_R} - 1\right)\frac{\rho^2}{\omega_z^2}\right)\cdot \tag{2}$$

$$\cdot \exp(-i(2p + |l| + 1)\varphi_g(z))\exp(il\phi)$$

Лагерр-Гауссовы моды

$$LG_{pl}(\rho,\phi,z) = \frac{N_{pl}}{\omega_z}\left(\frac{\rho\sqrt{2}}{\omega_z}\right)^{|l|} L_p^{|l|}\left(\frac{2\rho^2}{\omega_z^2}\right)\exp\left(\left(i\frac{z}{z_R}-1\right)\frac{\rho^2}{\omega_z^2}\right)\cdot$$

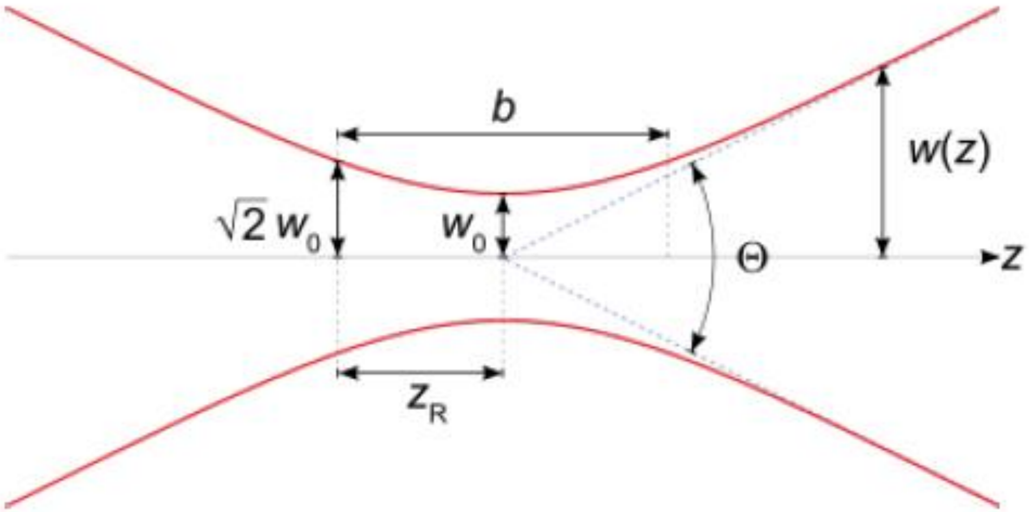
$$\cdot\exp(-i(2p+|l|+1)\varphi_g(z))\exp(il\phi)$$

$$\omega_z = \omega_0\sqrt{1+\frac{4z^2}{k^2\omega_0^4}} \text{ — радиус пучка в точке } z; \tag{3}$$

$$z_R = \frac{1}{2}k\omega_0^2 \text{ — длина Рэлея;} \tag{4}$$

$$\varphi_g = \arctan\left(\frac{z}{z_R}\right) \text{ — фаза Гуи;} \tag{5}$$

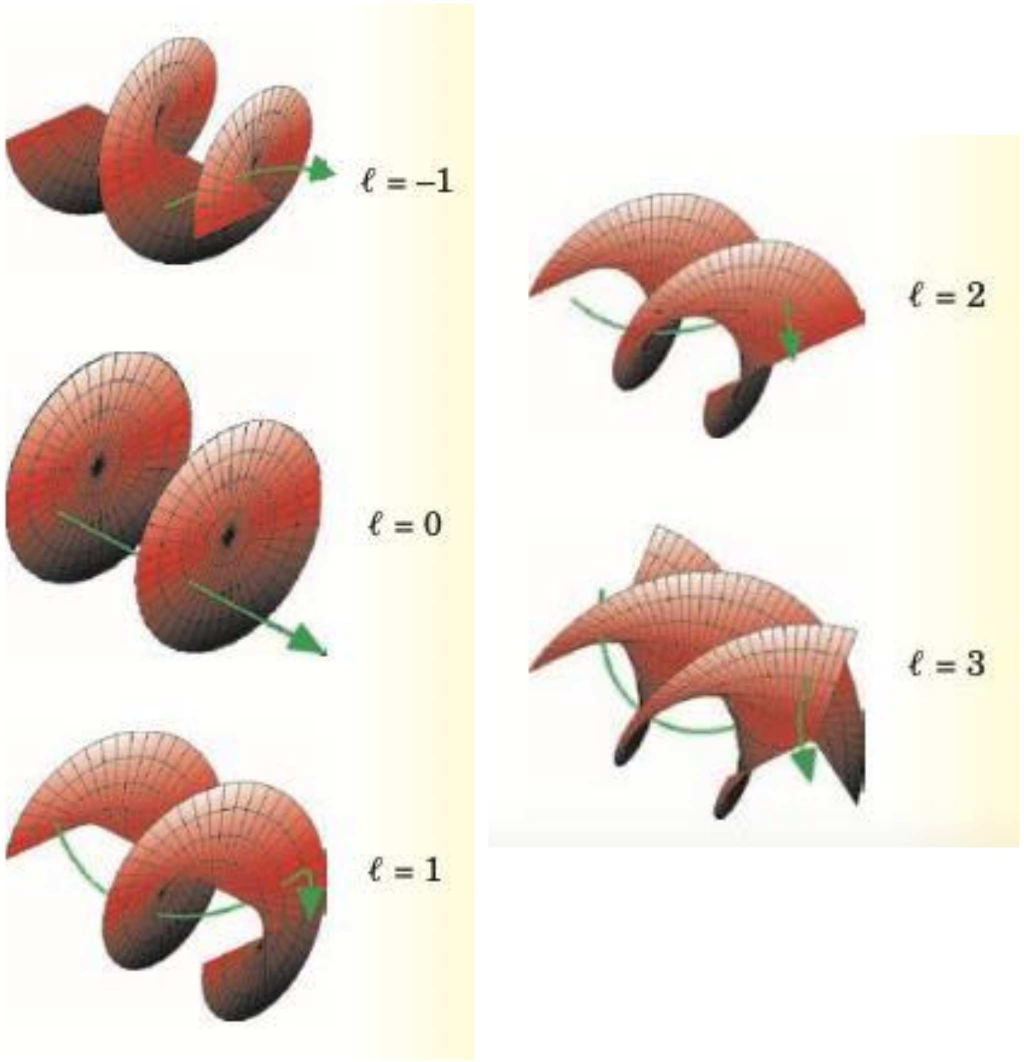
$$L_p^{|l|}(x) = \frac{e^x x^{-|l|}}{p!}\frac{d^p}{dx^p}\left(e^{-x}x^{p+|l|}\right) \tag{6}$$



Оператор ОУМ

$$\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}, \tag{7}$$

$$\hat{L}_z LG_{pl} = lLG_{pl} \tag{8}$$

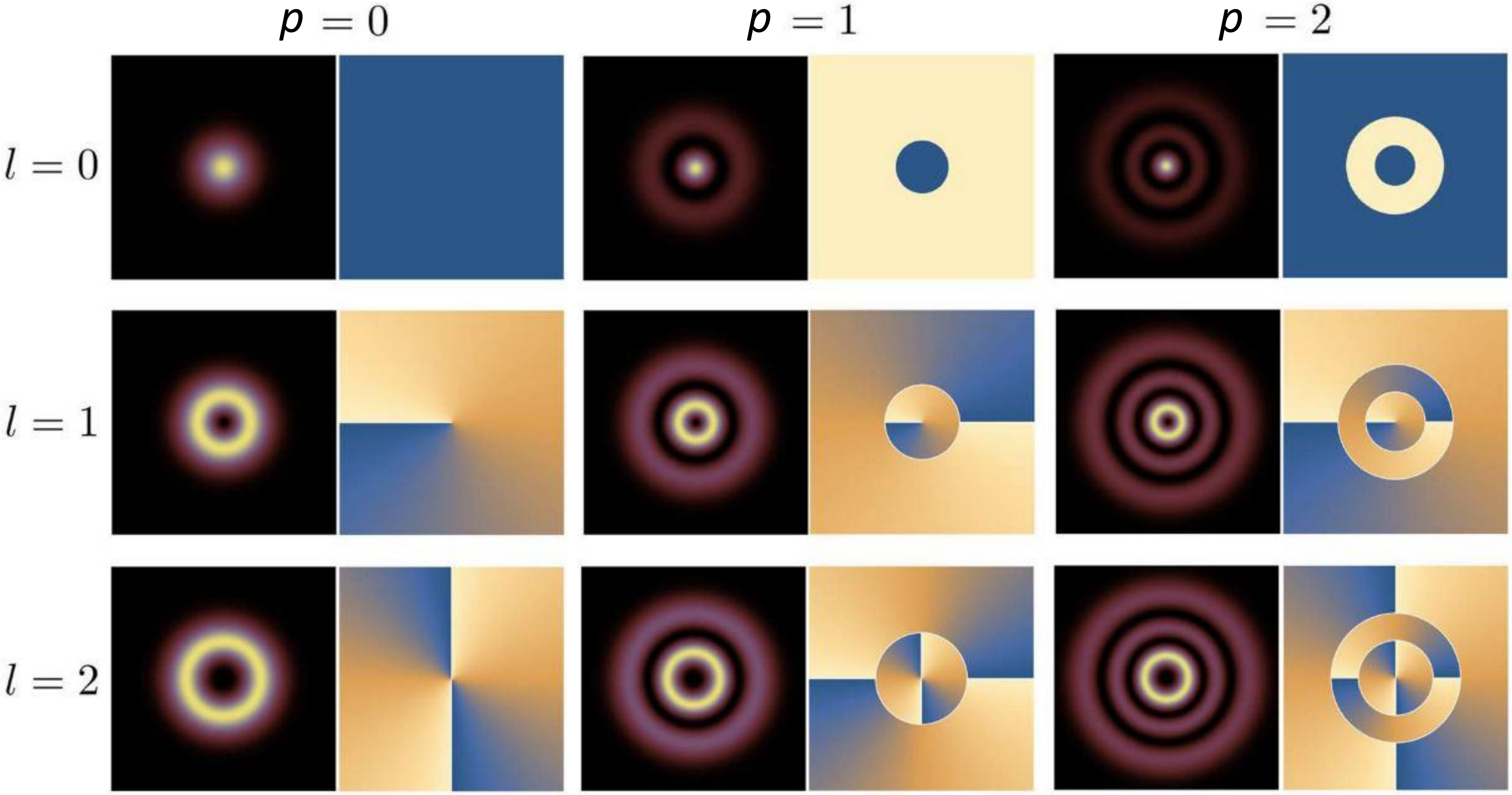


Оператор радиального индекса

$$\hat{\mathbb{P}}_z LG_{pl}(r, \phi, z) = pLG_{pl}(r, \phi, z)$$

$$\hat{\mathbb{P}}_z - ?$$

Профили фазы и интенсивности



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Radial quantum number of Laguerre-Gauss modes

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We introduce an operator linked with the radial index in the Laguerre-Gauss modes of a two-dimensional harmonic oscillator in cylindrical coordinates. We discuss ladder operators for this variable, and confirm that they obey the commutation relations of the su(1,1) algebra. Using this fact, we examine how basic quantum optical concepts can be recast in terms of radial modes.

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Physical meaning of the radial index of Laguerre-Gauss beams

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The Laguerre-Gauss modes are a class of fundamental and well-studied optical fields. These stable shape-invariant photons, exhibiting circular-cylindrical symmetry, are familiar from laser optics, micromechanical manipulation, quantum optics, communication, and foundational studies in both classical optics and quantum physics. They are characterized, chiefly, by two mode numbers: the azimuthal index indicating the orbital angular momentum of the beam, which itself has spawned a burgeoning and vibrant subfield, and the radial index, which up until recently has largely been ignored. In this paper we develop a differential operator formalism for dealing with the radial modes in both the position and momentum representations and, more importantly, give the meaning of this quantum number in terms of a well-defined physical parameter: the intrinsic hyperbolic momentum charge.

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Вращающиеся бозонные операторы

$$\hat{a}_{\pm} = \frac{1}{\sqrt{2}}(\hat{a}_x \mp \hat{a}_y) \tag{9}$$

Оператор полного числа квазичастиц:

$$\hat{n} = \hat{n}_{+} + \hat{n}_{-} \tag{10}$$

Оператор ОУМ:

$$\hat{l} = -i\frac{\partial}{\partial\phi} = \hat{n}_{+} - \hat{n}_{-} \tag{11}$$

$$[\hat{l}, \hat{n}] = 0 \rightarrow \{|n_{+}\rangle, |n_{-}\rangle\} - \text{общие собственные функции,} \tag{12}$$

тогда

$$n = n_{+} + n_{-}, l = n_{+} - n_{-} - \text{собственные значения операторов} \tag{13}$$

$$\langle r, \phi | n, l \rangle = \Psi_{nl}(r, \phi) = A_{nl}(r) e^{il\phi}, \tag{14}$$

$$A_{nl}(r, \phi) = \left(\frac{2\alpha p!}{(p + |l|)!} \right)^{\frac{1}{2}} e^{-\frac{\alpha^2 r^2}{2}} (\alpha r)^{|l|} \cdot L_p^{|l|}(\alpha^2 r^2) \tag{15}$$

$|\Psi_{nl}(r, \phi)|^2$—распределение вероятности появления p темных колец, (16)

$$p = \frac{1}{2}(n - |l|) \tag{17}$$

Введем для p оператор:

$$\hat{p} = \frac{1}{2}(\hat{n} - |\hat{l}|) = \begin{cases} \hat{n}_- & l > 0 \\ \hat{n}_+ & l < 0 \end{cases} \tag{18}$$

$$\hat{\mathbb{P}}_z = -\frac{\omega_z^2}{8}\nabla_\perp^2 + \frac{iz}{k\omega_z^2}\frac{\partial}{\partial r}r - \frac{\hat{l}}{2} + \frac{1}{2}\left(\frac{r^2}{\omega_z^2} - 1\right) \tag{21}$$

$$\hat{\mathbb{P}}_z = \frac{1}{2}\left(\hat{n} - \frac{1}{\hbar}|\hat{L}_z| + z^2\frac{1}{\hbar\omega}\frac{1}{2}\hat{p}_\perp^2 - z\frac{1}{\hbar}\hat{p}_\rho\right), \tag{22}$$

$$z \leftarrow \frac{z}{z_R}, \rho \leftarrow \frac{\hbar}{z_R c}\rho^2, [\frac{\hbar}{z_R c}] = 1 \tag{23}$$

$$\hat{n} = \hat{n_+} + \hat{n}_- \tag{24}$$

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \phi} = \hbar \hat{l} = \hbar(\hat{n}_+ - \hat{n}_-) \tag{25}$$

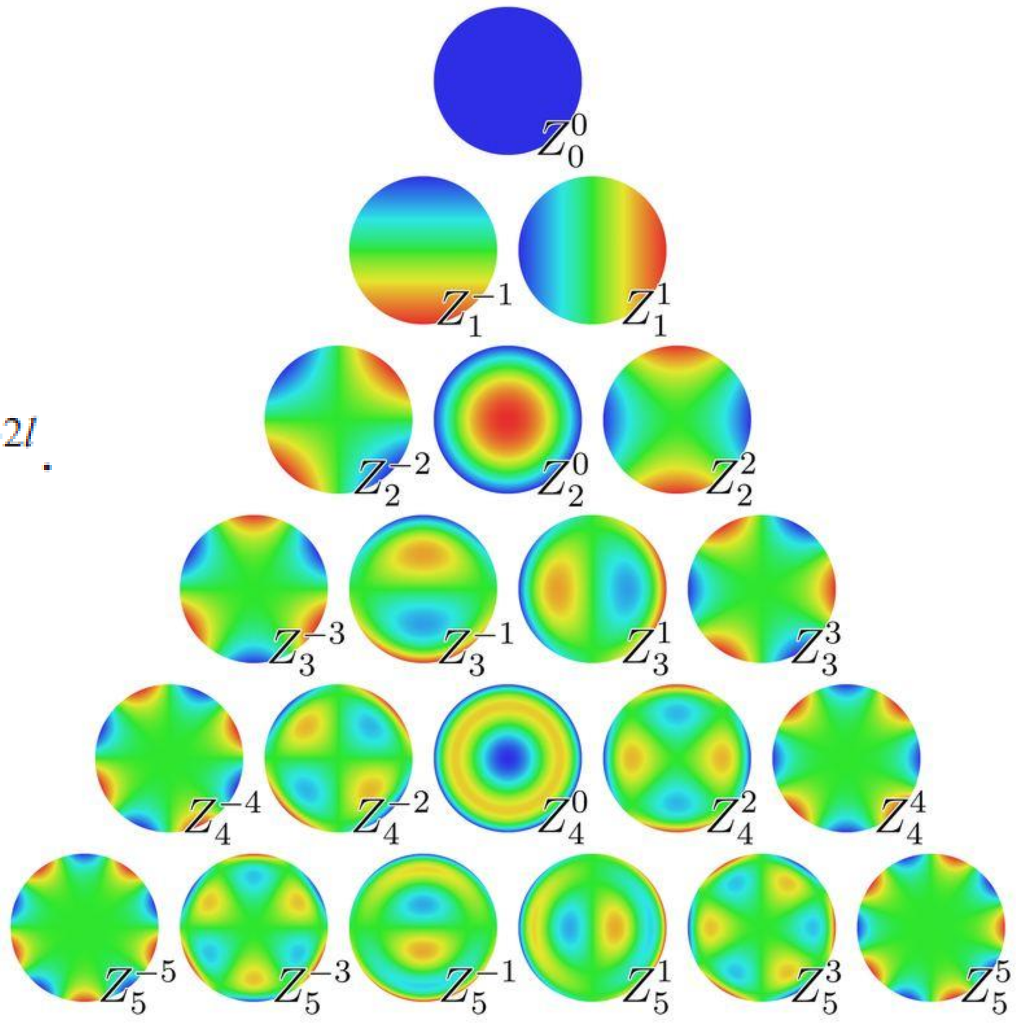
$$\hat{\mathbb{P}}_z = \frac{1}{2}\left(\hat{n} - |\hat{l}|\right) + z^2 \frac{1}{\hbar\omega} \frac{1}{2} p_\perp^2 - z \frac{1}{\hbar} p_\rho \tag{26}$$

Полиномы Цернике

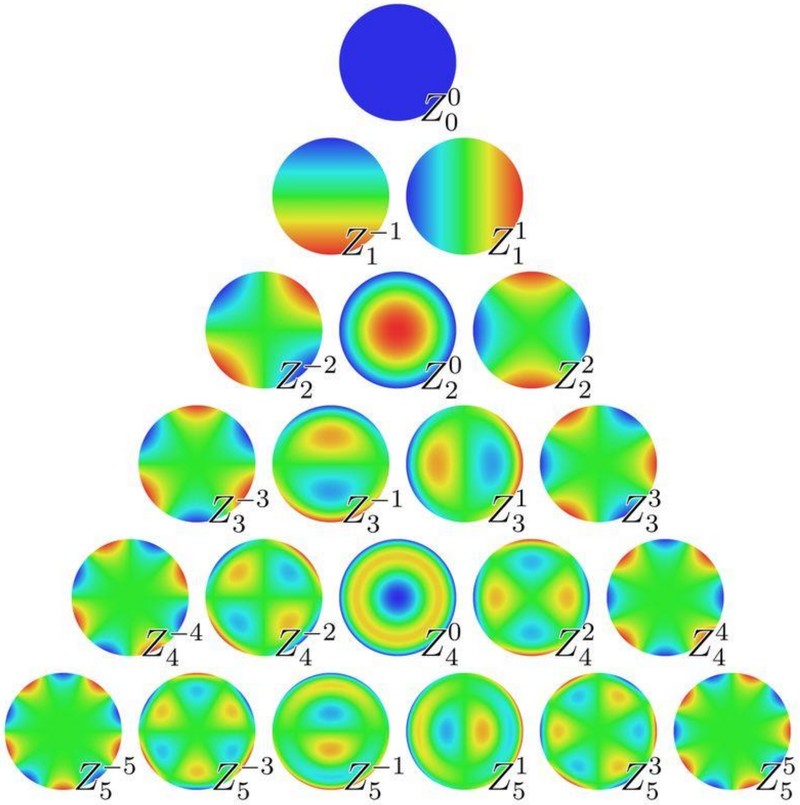
$$Z_n^m(r,\theta) = R_n^m(r)\cos m\theta \quad \text{for } m \geq 0,$$
$$Z_n^{-m}(r,\theta) = R_n^m(r)\sin m\theta \quad \text{for } m < 0,$$

$$R_n^m(r) = \sum_{l=0}^{(n-m)/2} \frac{(-1)^l (n-l)!}{l!\left[\frac{1}{2}(n+m)-l\right]!\left[\frac{1}{2}(n-m)-l\right]!} r^{n-2l}.$$

n=m+2p, p-целое число



Полиномы Цернике

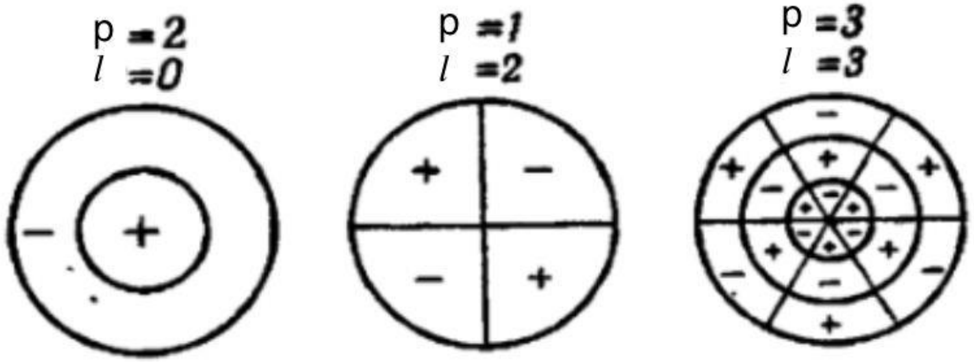


$$E(r,\phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} C_{nm} \Psi_{nm}(r,\phi),$$

$$\Psi_{nm}(r,\phi) = A_{nm} \cdot R_n^m(r) \cdot \exp\left(im\phi\right)$$

$$C_{nm} = 2\pi \cdot A_n^m \sqrt{p!(p+|l|)!} \cdot \sum_{i=0}^{p} \sum_{h=0}^{(n-m)/2} (-1)^{i+h} \cdot \frac{(n-h)!}{(p-i)!(|l|+i)!i!(\frac{1}{2}(n+|l|)-h)!(\frac{1}{2}(n-|l|)-h)!h!} \cdot \left(\frac{\omega_0^2}{2R^2}\right)^{\frac{1}{2}(n-2h)} \cdot \gamma(i-n+\frac{1}{2}(|l|+n)+1, \frac{2R^2}{\omega_0^2}) \quad (15)$$

Стоячая волна круглой мембраны



Фиг. 53. Некоторые формы колебаний круглой мембраны, закрепленной по краю.

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